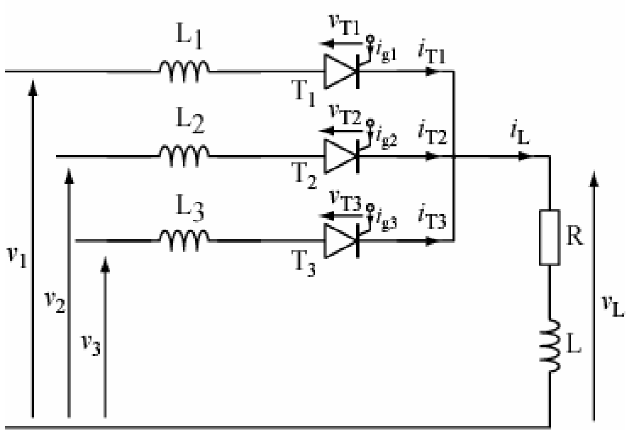
Lecture 4
Converters II - Overlap

Objectives

- Finite inductance in AC supply lines causes overlap in which case transfer of load current from one device to another in converter does not take place instantaneously
- We determine expressions for duration of overlap and calculate effect of overlap on mean load voltage of converter
- We model effect of overlap on converter output voltage by simple circuit consisting of ideal voltage source and resistor, whose value depends only on AC supply inductance and supply frequency

Introduction

- 3-phase, half-wave bridge with source inductance in AC supply lines
  - Consider instant of firing thyristor T_2 with T_1 conducting:



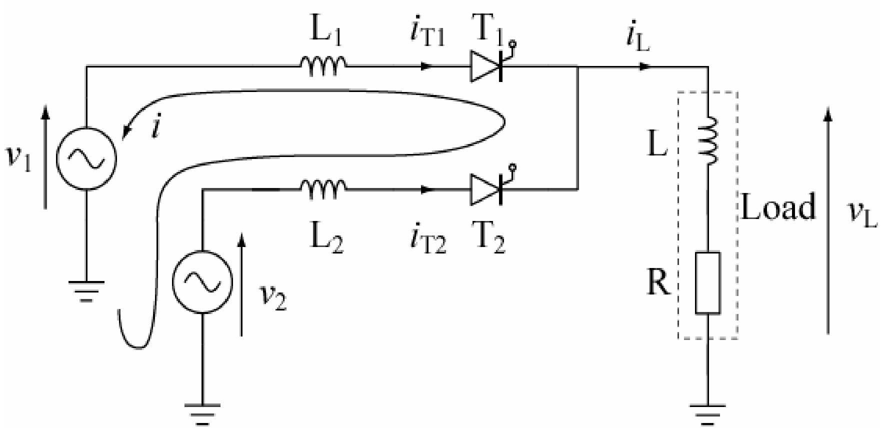
- When T_2 is fired, current I_{T1} can not fall immediately to zero and I_{T2} can not increase to equal I_L immediately.
- This is because voltage and current of inductor are related according to:

$$V_L = L\frac{dI_L}{dt}$$

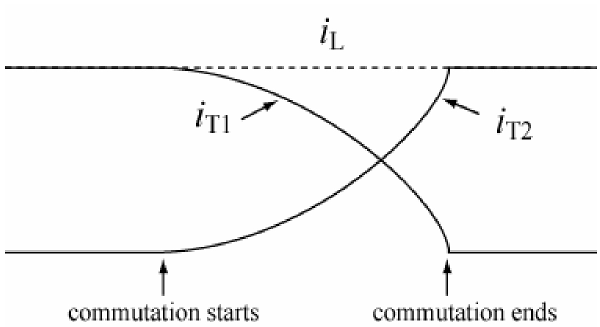
- Infinite dI_L/dt requires infinite voltage: fastest dI_L/dt limited by supply voltage
- Load current switches from T_1 to T_2 in finite time period with both T_1 and T_2 are conducting
- Overlap affects load voltage waveform and hence mean load voltage
- In practice, supply inductance < 1 mH; sufficient to have significant effect

Definition of overlap parameters

- Consider transfer of load current from T_1 to T_2
- AC supply V_3 and thyristor T_3 are not involved in this transfer of current
- Show AC supplies V_1 to V_2 with their inductance L, T_1, T_2 and load:



- We assume a constant load current I_L
- Transfer of load current from T_1 to T_2 during switching:



- Actual dynamic currents = initial conditions ($I_{LT1} = I_L$, $I_{T2} = 0$) + current i circulates between T_1 and T_2 and AC supply sources

- Circulating current i increases from zero at instant of firing T_2 until it equals load current I_L; at this point current I_{T1} has become zero when this thyristor is turned off
- Interval when both thyristors are conducting is referred to as overlap interval defined by overlap angle γ

Determining the overlap angle γ

- During overlap interval both thyristors considered as short circuits
- Circuit consists of 2 voltage sources V_1 and V_2 and 2 inductances L

$$v_2 - v_1 = 2L\frac{di}{dt}$$

- Difference between phase voltages $v_2 - v_1$ is equal to line votages V_{21}:

$$v_2 - v_1 = V_{21} = \sqrt{3}V_m \sin(\omega t + \alpha)$$

- $t = 0$ corresponds to instant thyristor T_2 is fired ($\theta = \alpha$)

$$di = \frac{\sqrt{3}V_m}{2L}\sin(\omega t + \alpha)dt$$

- Integrate equation from t = 0 to t:

$$i = \int_0^t di = \int_0^t \frac{\sqrt{3}V_m}{2L}\sin(\omega t + \alpha)dt$$

- Subsitute $\theta = \omega t + \alpha$

$$i = \frac{\sqrt{3}V_m}{2\omega L}\int_{\alpha}^{\omega t+\alpha}\sin\theta d\theta \; = \frac{\sqrt{3}V_m}{2\omega L}\left[-\cos\theta\right]_{\alpha}^{\omega t+\alpha}$$

$$= \frac{\sqrt{3}V_m}{2\omega L}\left(\cos\alpha - \cos(\omega t + \alpha)\right)$$

- Overlap interval is complete when $i = I_L$ and $\omega t = \gamma$

$$\mathrm{I}_L = \frac{\sqrt{3}V_m}{2\omega L}\left[\cos\alpha - \cos(\gamma + \alpha)\right]$$

or

$$\gamma = \cos^{-1}\left[\cos\alpha - \frac{2\omega L}{\sqrt{3}V_m}I_L\right] - \alpha$$

- Overlap angle depends on
  - Firing angle α
  - Supply frequency ω
  - Supply inductance L
  - Peak phase voltage of the supply V_m
  - Load current I_L

- If L = 0:

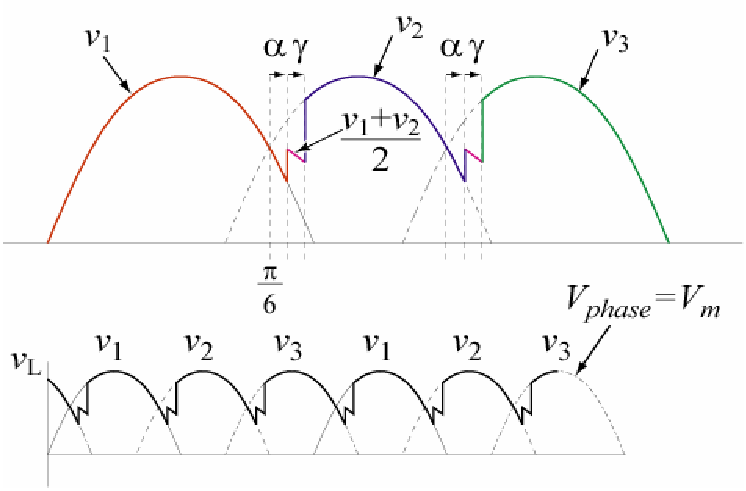
$$\gamma\big|_{L=0} = -\alpha + \cos^{-1}(\cos\alpha) = -\alpha + \alpha = 0$$

i.e. no overlap

- Same derivation describes the transfer of load current between T_2 and T_3 and between T_3 and T_1

Mean load voltage with overlap

- During overlap both T_1 and T_2 conduct
- Load voltage = mean of phase voltages v_1 and v_2:



  - 3 phase voltages and load voltage for 3-phase half-wave bridge converter with overlap for $\alpha \approx 20^o$ and $\gamma \approx 20^o$

- $\theta = \pi/6$: Vertical line shows instant at which v_1 and v_2; instant at which diode replacing T_2 would conduct $\therefore$ reference for determining α
- $\theta = \pi/6 + \alpha$: T_2 fired; load voltage follows mean of v_1 and v_2
- $\theta = \pi/6 + \alpha + \gamma$: overlap completed; load voltage follows

- Mean load voltage of ideal converter with no overlap:

$$V_{mean} = \frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\theta d\theta$$

- $\theta = \pi/6 + \alpha$ until $\theta = \pi/6 + \alpha + \gamma$, load voltage = mean of v_1 and v_2
- $\theta = \pi/6 + \alpha + \gamma$ until $\theta = 5\pi/6 + \alpha$, load voltage = v_2
- And we can express v_1 and v_2 as

$$v_1 = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) \qquad v_2 = V_m \sin \omega t$$

- Thus, the mean voltage is,

$$V_{mean}=\frac{3}{2\pi}\left\{\frac{1}{2}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{6}+\alpha+\gamma} V_m\left[\sin\left(\theta+\frac{2\pi}{3}\right)+\sin\theta\right]d\theta+\int_{\frac{\pi}{6}+\alpha+\gamma}^{\frac{5\pi}{6}+\alpha} V_m\sin\theta d\theta\right\}$$

$$=\frac{3V_m}{2\pi}\left\{\int_{\frac{\pi}{6}+\alpha+\gamma}^{\frac{5\pi}{6}+\alpha}\sin\theta d\theta+\frac{1}{2}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{6}+\alpha+\gamma}\left[\sin\left(\theta+\frac{2\pi}{3}\right)+\sin\theta\right]d\theta\right\}$$

$$=\frac{3V_m}{2\pi}\left\{\left[-\cos\theta\right]_{\frac{\pi}{6}+\alpha+\gamma}^{\frac{5\pi}{6}+\alpha}+\frac{1}{2}\left[-\cos\left(\theta+\frac{2\pi}{3}\right)\right]_{\frac{5\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha+\gamma}+\frac{1}{2}\left[-\cos\theta\right]_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{6}+\alpha+\gamma}\right\}$$

$$=\frac{3\sqrt{3}V_m}{4\pi}\left(\cos(\alpha+\gamma)+\cos\alpha\right)$$

- Finally,

$$V_{mean}=\frac{3\sqrt{3}V_m}{4\pi}\left[\cos(\alpha+\gamma)+\cos\alpha\right]$$

- Notice if $L_1 = L_2 = L_3 = 0$, then $\gamma = 0$:

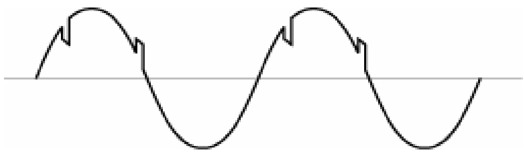
$$V_{mean(ideal)}=\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$$

as in ideal case

- During overlap intervals, pairs of thyristors are conducting
- This has effect on supply voltage at anodes of thyristors

e.g. voltage on anode of T_1 with overlap

- When conduction changes from T_1 and T_2 or from T_3 to T_1, v_1 becomes mean of v_1 and v_2 or v_1 and v_3, respectively, during overlap intervals



Equivalent circuit for the effect of overlap

$$V_{mean} = V_{mean(ideal)} - \Delta V_d$$

where ΔV_d is the change in converter output voltage because of overlap

- Hence we can determine the effect of overlap

$$\begin{aligned}\Delta V_d &= V_{mean(ideal)} - V_{mean} \\ &= \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha - \frac{3\sqrt{3}V_m}{4\pi}\left[\cos(\alpha+\gamma)+\cos\alpha\right] \\ &= \frac{3\sqrt{3}V_m}{4\pi}\left[\cos\alpha - \cos(\alpha+\gamma)\right]\end{aligned}$$

- α and γ related to load current I_L and other parameters:

$$I_L = \frac{\sqrt{3}V_m}{2\omega L}\left[\cos\alpha - \cos(\gamma + \alpha)\right]$$

- Effect of overlap:

$$\Delta V_d = \frac{3\sqrt{3}V_m}{4\pi}\frac{2\omega L I_L}{\sqrt{3}V_m}$$

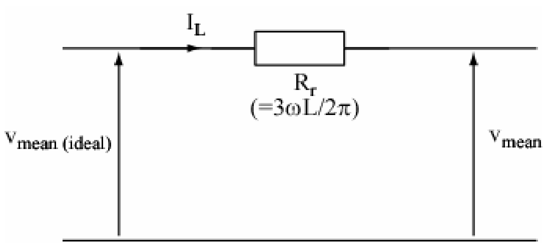
$$= \frac{3\omega L}{2\pi} I_L$$

- Mean load voltage with overlap:

$$V_{mean} = V_{mean(ideal)} - \Delta V_d$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha - \frac{3\omega L}{2\pi} I_L$$

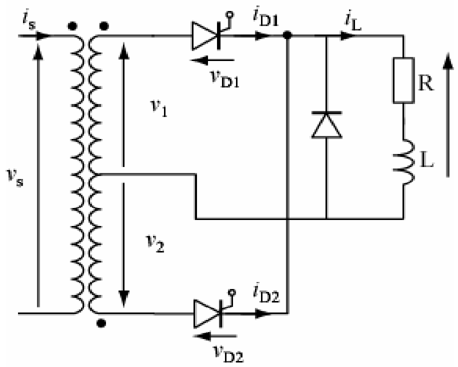
- Reduction in voltage modelled by series resistor:



- R_r models reduction of mean load voltage due to overlap but it does not correctly model power loss because overlap phenomenon is lossless

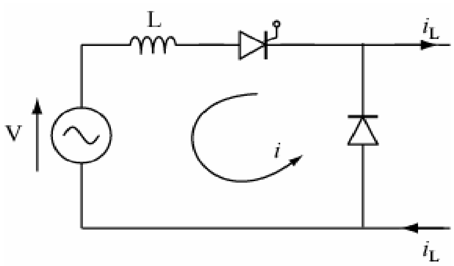
Overlap example

- Single-phase, full-wave converter with a freewheeling diode is supplied from a 120 V, 50 Hz AC supply with a source inductance of 0.32 mH



  - Assuming that load current is constant at 4 A and firing angle α = 15º
  - Find overlap angles for:
    - (i) transfer of current from conducting thyristor to diode
    - (ii) transfer of current from commutating diode to a thyristor when the firing angle is 15º

- *Solution*
  - (i) Commutation from thyristor to diode begins when load voltage starts to reverse which we define as the reference instant t = 0; assume device forward voltage drops are zero
  - Represent transfer of current from initially conducting thyristor to freewheeling diode by means of circulating current i:



- During overlap, both devices are conducting:

$$v = -V_m \sin \omega t = -L \frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t dt$$

$$i = \frac{V_m}{L} \int_0^t \sin \omega t dt$$

$$= \frac{V_m}{\omega L} (1 - \cos \omega t)$$

- Commutation complete when $i = I_L$, at which point $\omega t = \gamma_1$; thus

$$I_L = \frac{V_m}{\omega L} (1 - \cos \gamma_1)$$

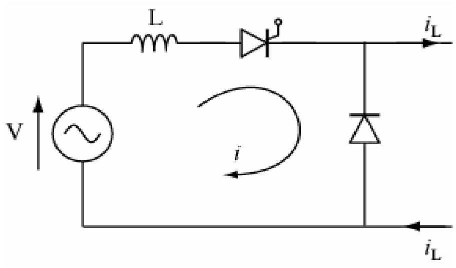
- Hence,

$$\gamma_1 = \cos^{-1} \left[1 - \frac{\omega L I_L}{V_m} \right]$$

- Substitution of values for ω, L, I_L, and V_m, we obtain:

$$\gamma_1 = 3.95^\circ$$

- (ii) Commutation from diode to thyristor begins at instant of firing thyristor; introduce circulating current i to represent change in currents:



- Before thyristor is fired, load current flows through diode
- During overlap interval, both devices are conducting and we can write:

$$v = V_m \sin(\omega t + \alpha) = L\frac{di}{dt}$$

- From which, we have:

$$di = \frac{V_m}{L}\sin(\omega t + \alpha)dt$$

- Integrating from 0 to t:

$$i = \frac{V_m}{\omega L}\left[\cos\alpha - \cos(\omega t + \alpha)\right]$$

- Commutation complete when $i = I_L$, at which point $\omega t = \gamma_2$; thus:

$$I_L = \frac{V_m}{\omega L}\left[\cos\alpha - \cos(\gamma_2 + \alpha)\right]$$

- Hence:

$$\gamma_2 = \alpha + \cos^{-1}\left[\cos\alpha - \frac{\omega L I_L}{V_m}\right]$$

- Substitution of values for α, ω, L, I_L, and V_m, we obtain:

$$\gamma_2 = 0.516^\circ$$

Summary

- Have shown that finite inductance in AC supply lines in converter causes overlap in which case transfer of load current from one device to another does not take place instantaneously
- Have determined expressions for duration of overlap in converter and its effect on mean load voltage
- Have also modelled effect of overlap on converter output voltage by simple consisting of ideal voltage source and resistor
- Next, we consider operation of fully controlled converters in inverter mode