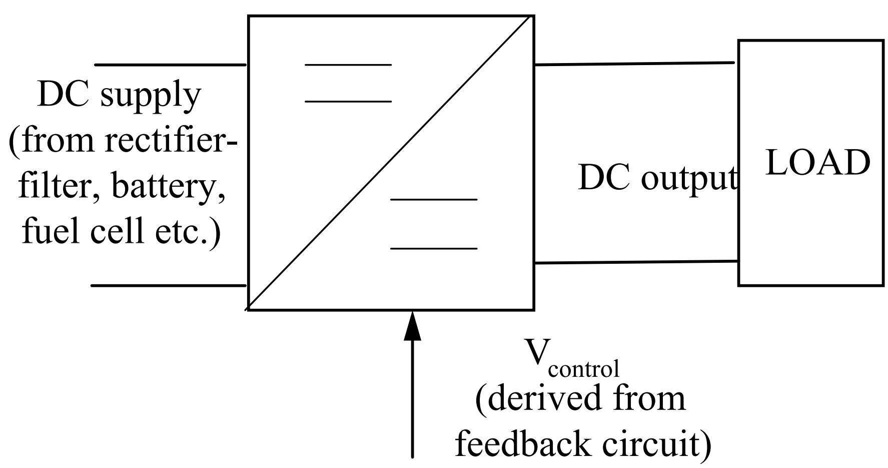
Chapter 3
DC to DC CONVERTER (CHOPPER)

- Basic non-isolated DC-DC converter topologies: Buck, Boost, Buck-Boost, Cuk in CCM and DCM mode
- Non-ideal effects on converter performance
- Isolated DC-DC converters, switched-mode power supply
- Control of DC-DC converters
- High frequency transformer and inductor design
- Notes on electromagnetic compatibility (EMC) and solutions.

DC-DC Converter (Chopper)

- DEFINITION: Converting the unregulated DC input to a controlled DC output with a desired voltage level.

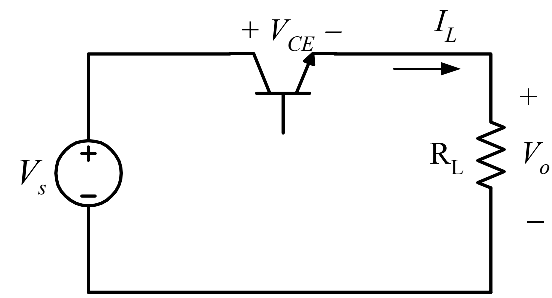
- General block diagram:



- APPLICATIONS:
  - Switched-mode power supply (SMPS), DC motor control, battery chargers

Linear regulator

- Transistor is operated in linear (active) mode.

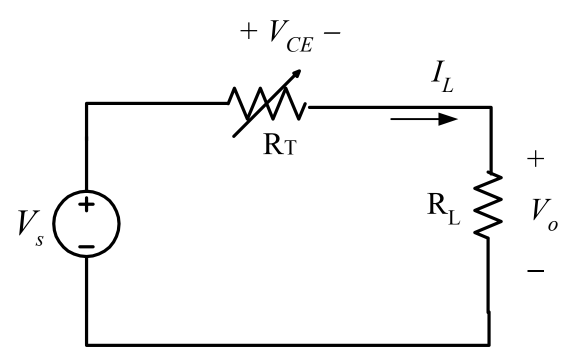


MODEL OF LINEAR REGULATOR

- Output voltage

$$V_o = I_L R_T$$

- The transistor can be conveniently modelled by an equivalent variable resistor, as shown.



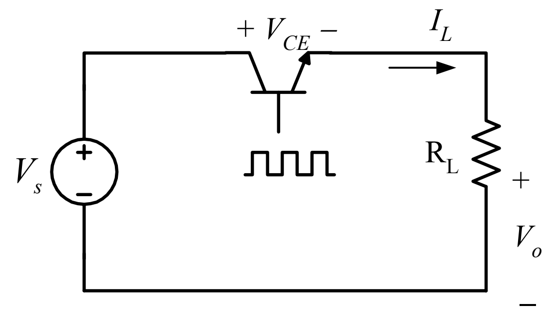
EQUIVALENT CIRCUIT

- Power loss is high at high current due to:

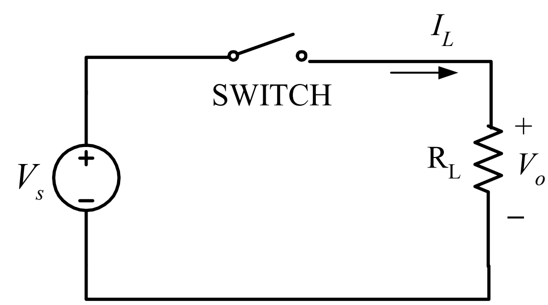
$$P_o = I_L^2 R_T$$

Switching Regulator

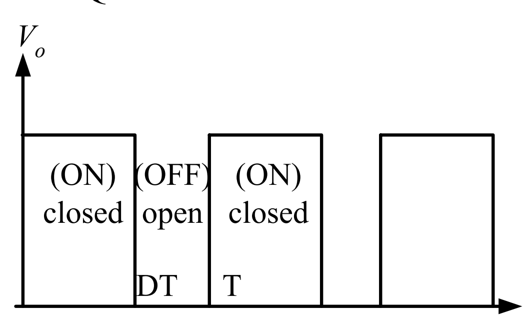
- Power loss is **zero** (for ideal switch):
  - when switch is open, no current flow in it,
  - when switch is closed no voltage drop across it.
  - Since power is a product of voltage and current, no losses occurs in the switch.
  - Power is 100% transferred from source to load.



MODEL OF LINEAR REGULATOR



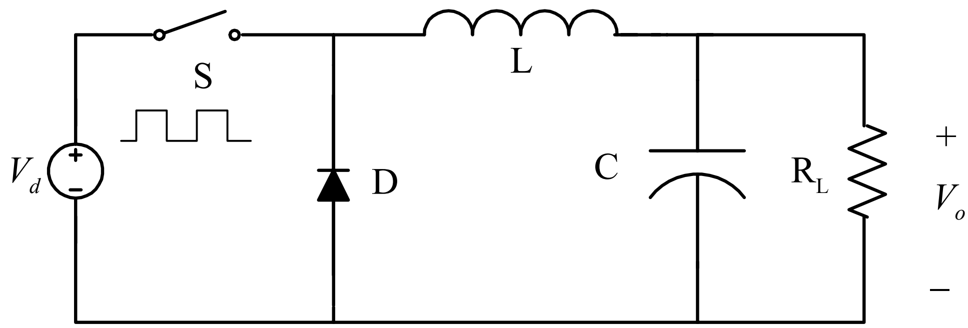
EQUIVALENT CIRCUIT



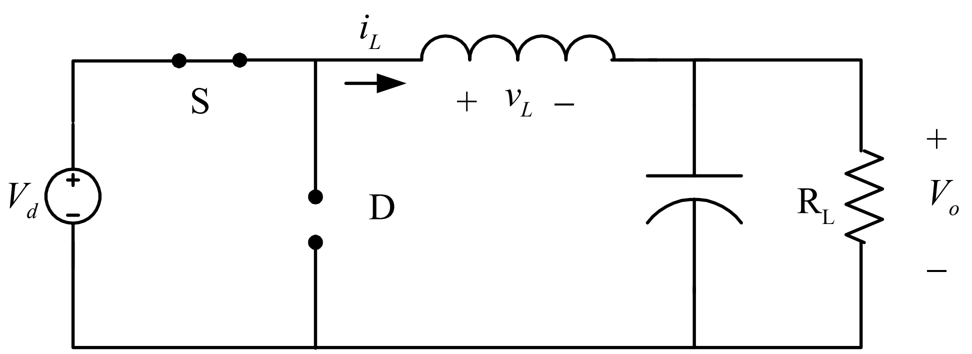
OUTPUT VOLTAGE

- Switching regulator is the basis of all DC-DC converters

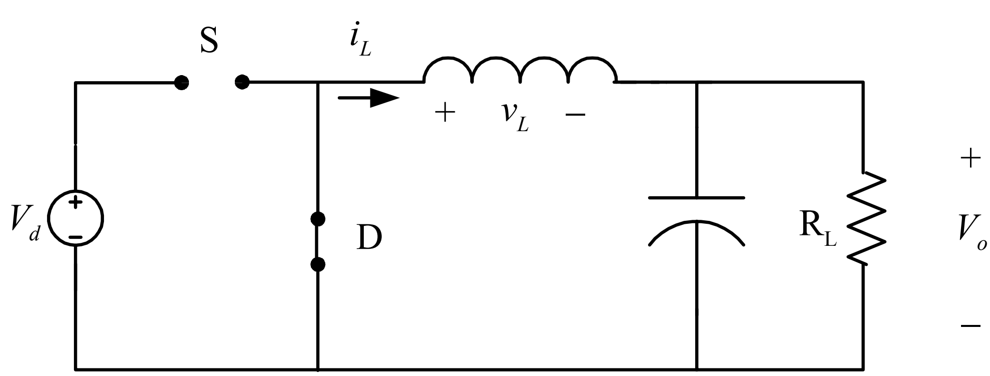
Buck (step-down) converter



CIRCUIT OF BUCK CONVERTER



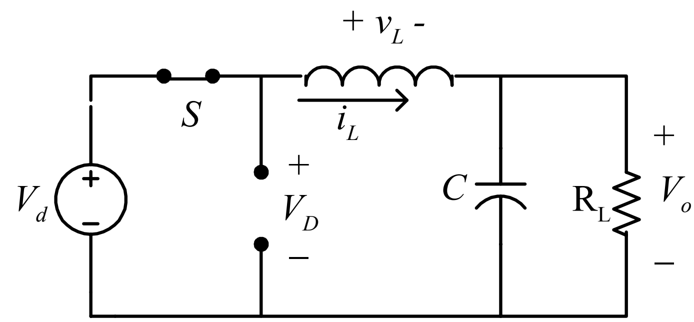
CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED

Circuit operation when switch is turned on (closed)

- Diode is reversed biased. Switch conducts inductor current



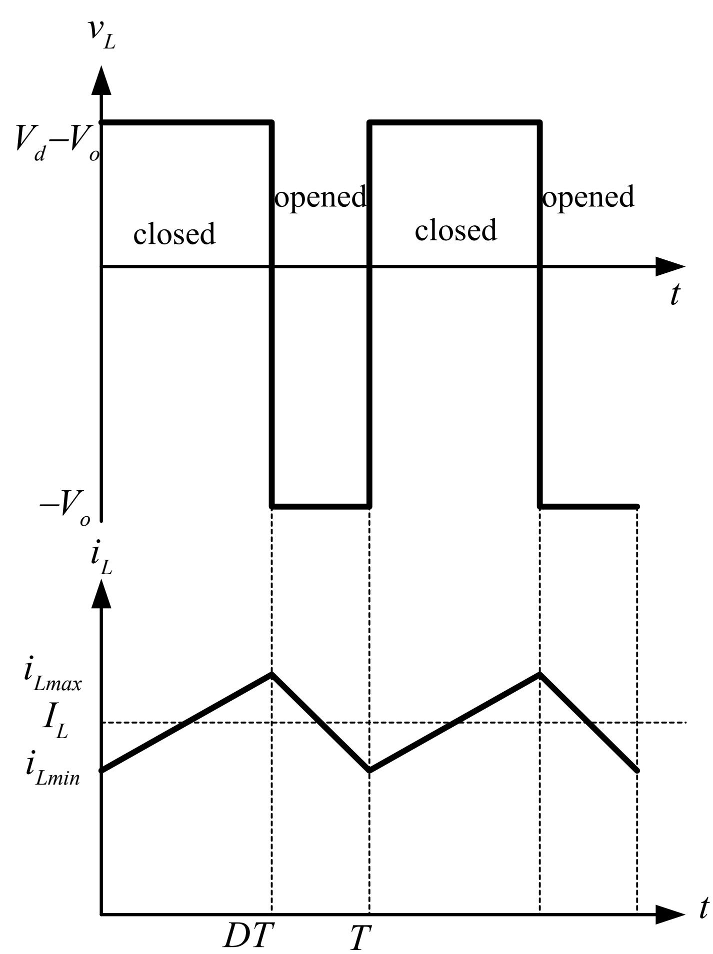
- This results in positive inductor voltage, i.e:

$$v_L = V_d - V_o$$

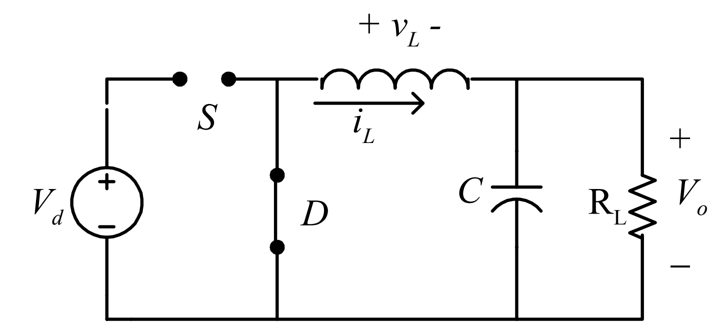
- It causes linear increase in the inductor current

$$v_L = L\frac{di_L}{dt}$$

$$\Rightarrow i_L = \frac{1}{L}\int v_L dt$$

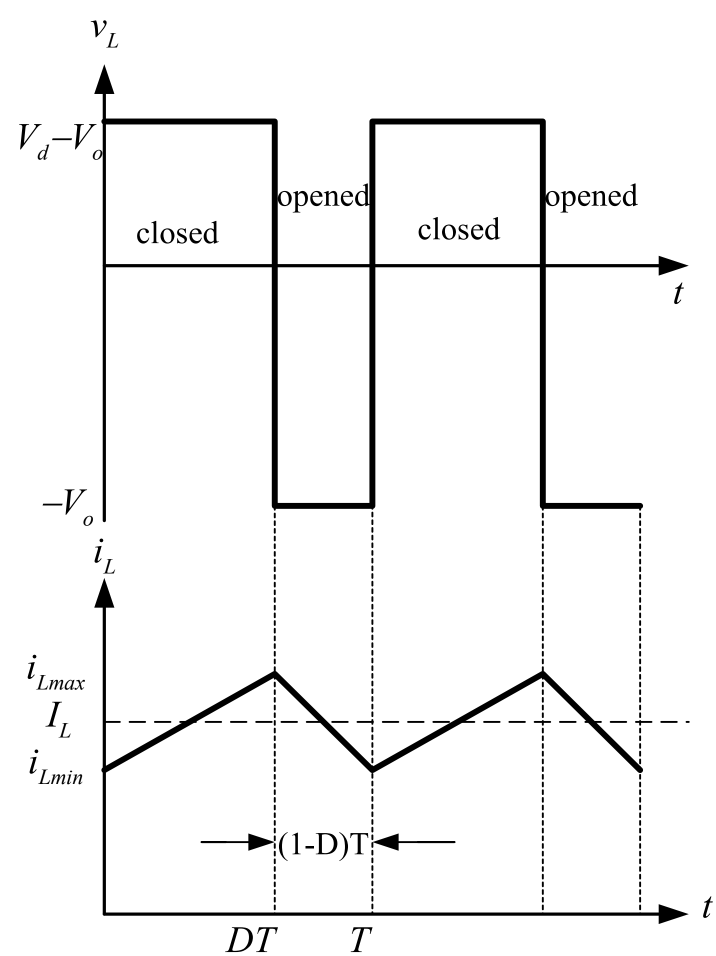


Operation when switch turned off (opened)



- Because of inductive energy storage, i_L continues to flow.
- Diode is forward biased
- Current now flows through the diode and

$$v_L = -V_o$$

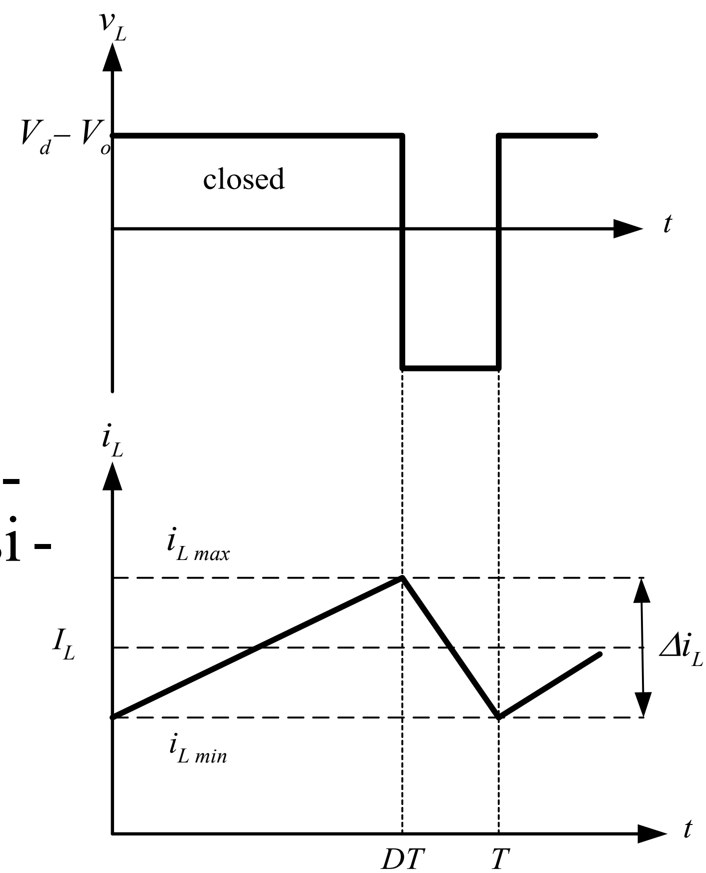


Analysis for switch closed

The inductor voltage,

$$v_L = V_d - V_o$$
$$= L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$



Note: since the derivative of i_L is a positive constant. Therefore i_L must increase linearly.

From Figure

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_d - V_o}{L}$$

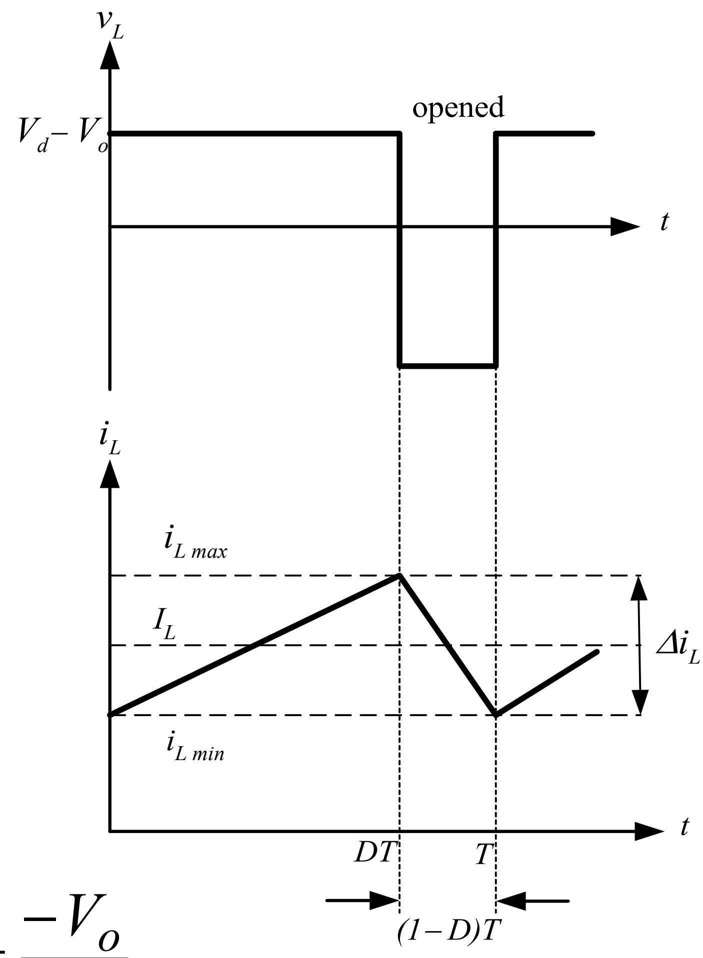
$$(\Delta i_L)_{closed} = \left(\frac{V_d - V_o}{L}\right) \cdot DT$$

Analysis for switch opened

For switch opened,

$$v_L = -V_o$$
$$= L\frac{di_L}{dt}$$
$$\Rightarrow \frac{di_L}{dt} = \frac{-V_o}{L}$$

Note : since the derivative of i_L is a negative constant, i_L must decrease linearly.

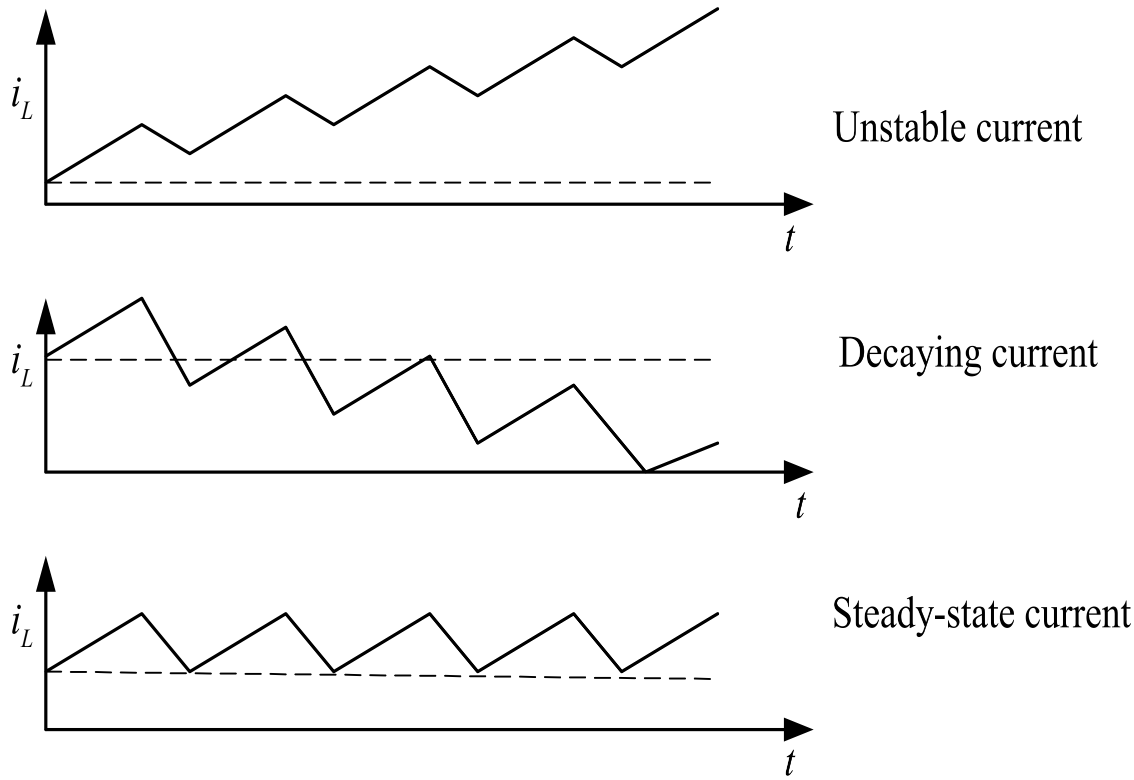


From Figure

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{-V_o}{L}$$

$$(\Delta i_L)_{opened} = \left(\frac{-V_o}{L}\right)\cdot(1-D)T$$

Steady-state operation



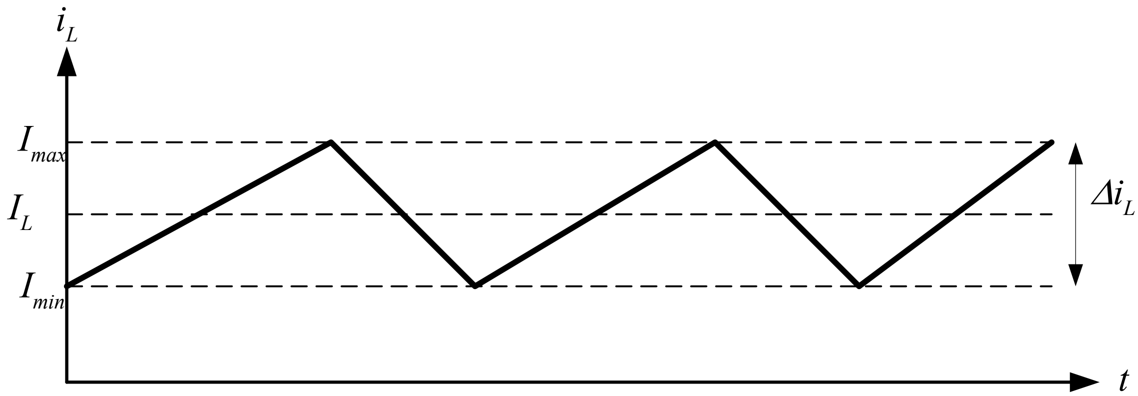
Steady-state operation requires that i_L at the end of switching cycle is the same at the begining of the next cycle. That is the change of i_L over one period is zero, i.e :

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\left(\frac{V_d - V_o}{L}\right) \cdot DT_s - \left(\frac{-V_o}{L}\right) \cdot (1-D)T_s = 0$$

$$\Rightarrow V_o = DV_d$$

Average, Maximum and Minimum inductor current



Average inductor current = Average current in R_L

$$\Rightarrow I_L = I_R = \frac{V_o}{R}$$

Maximum current :

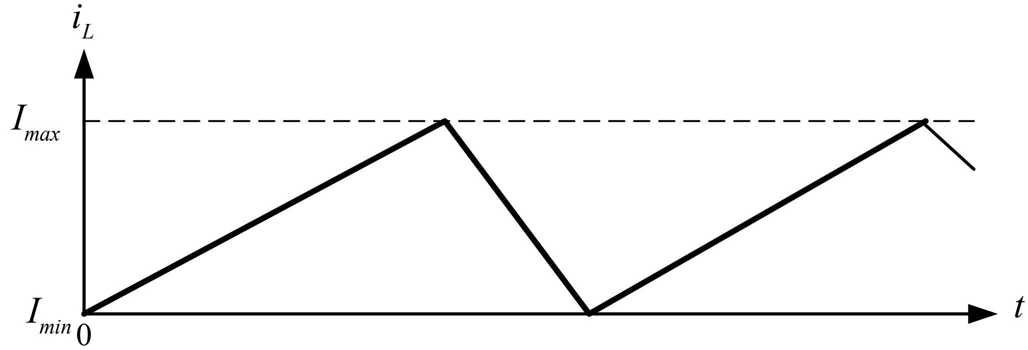
$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{1}{2}\left(\frac{V_o}{L}(1-D)T\right)$$

$$= V_o\left(\frac{1}{R} + \frac{(1-D)}{2Lf}\right)$$

Minimum current :

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = V_o\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right)$$

Continuous current operation



From previous analysis,

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = V_o\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right)$$

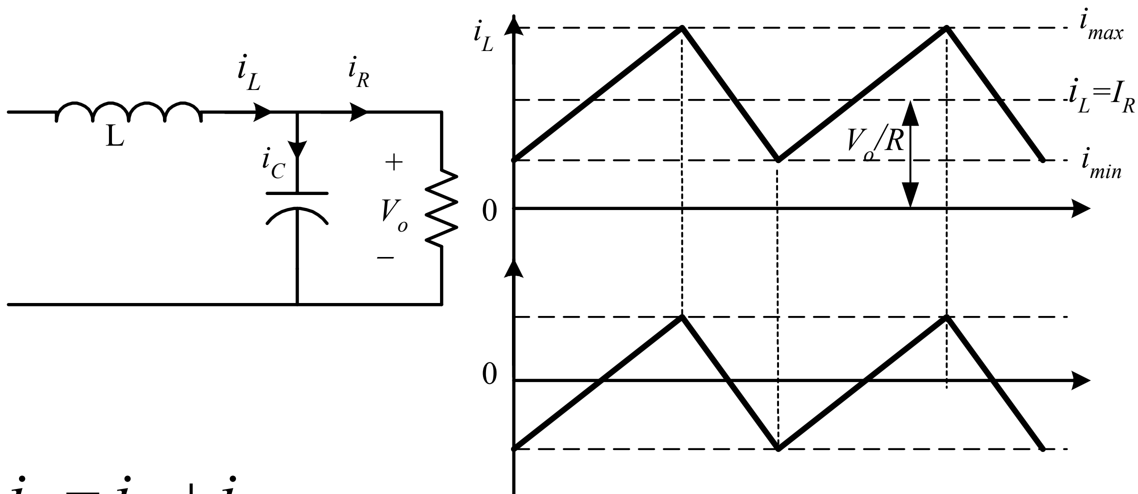
For continuous operation, $I_{\min} \geq 0$,

$$\Rightarrow V_o\left(\frac{1}{R} - \frac{(1-D)}{2Lf}\right) \geq 0$$

$$\Rightarrow L \geq L_{\min} = \frac{(1-D)}{2f} \cdot R$$

This is the minimum inductor current to ensure continous mode of operation. Normally L is chosen be be >> $L_{\min}$

Output voltage ripple



$$i_c = i_L + i_R$$

$$Q = CV_o \Rightarrow \Delta Q = C\Delta V \Rightarrow_o \Delta V_o = \frac{\Delta Q}{C}$$

From figure, use triangle area formula :

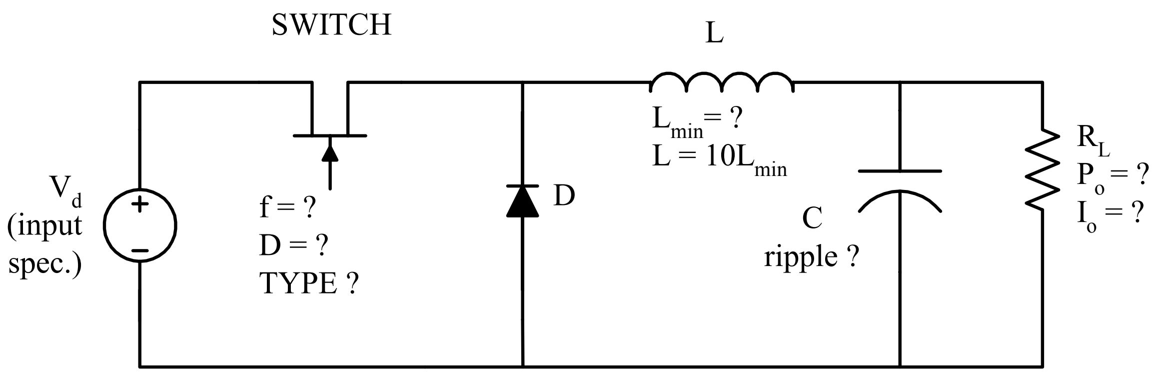
$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8}$$

$$\therefore \Delta V_o = \frac{T\Delta i_L}{8C} = \frac{(1-D)}{8LCf^2}$$

So, the ripple factor,

$$r = \frac{\Delta V_o}{V_o} = \frac{(1-D)}{8LCf^2}$$

Design procedures for Buck



- Calculate D to obtain required output voltage.

- Select a particular switching frequency:
  - preferably >20KHz for negligible acoustic noise
  - higher fs results in smaller L, but higher device losses. Thus lowering efficiency and larger heat sink. Also C is reduced.
  - Possible devices: MOSFET, IGBT and BJT. Low power MOSFET can reach MHz range.

Design procedures for Buck

- Determine L_{min}. Increase L_{min} by about 10 times to ensure full continuos mode.

- Calculate C for ripple factor requirement.

- Capacitor ratings:
  - must withstand peak output voltage
  - must carry required RMS current. Note RMS current for triangular w/f is $I_p/3$, where I_p is the peak capacitor current given by $\Delta i_L/2$

- Wire size consideration:
  - Normally rated in RMS. But i_L is known as peak. RMS value for i_L is given as:

$$I_{L,RMS} = \sqrt{I_L^{\,2} + \left(\frac{\Delta i_L/2}{\sqrt{3}}\right)^2}$$

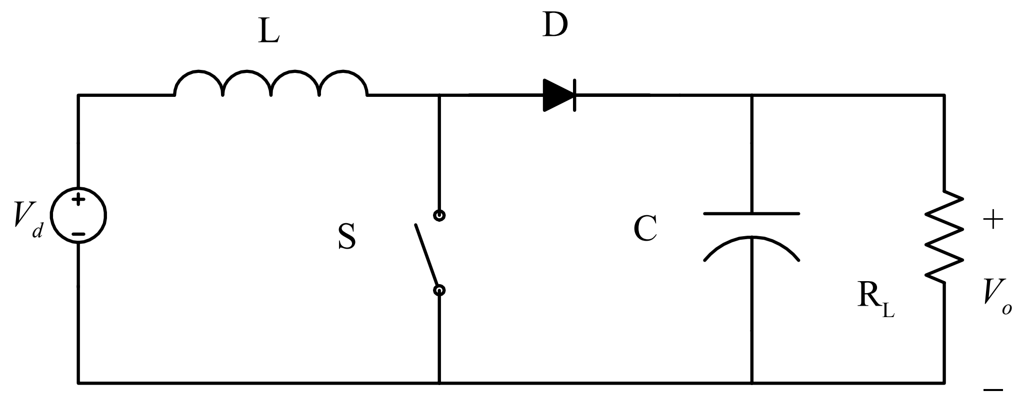
Examples of Buck converter

- A buck converter is supplied from a 50V battery source. Given L=400uH, C=100uF, R=20 Ohm, f=20KHz and D=0.4. Calculate: (a) output voltage (b) maximum and minimum inductor current, (c) output voltage ripple.

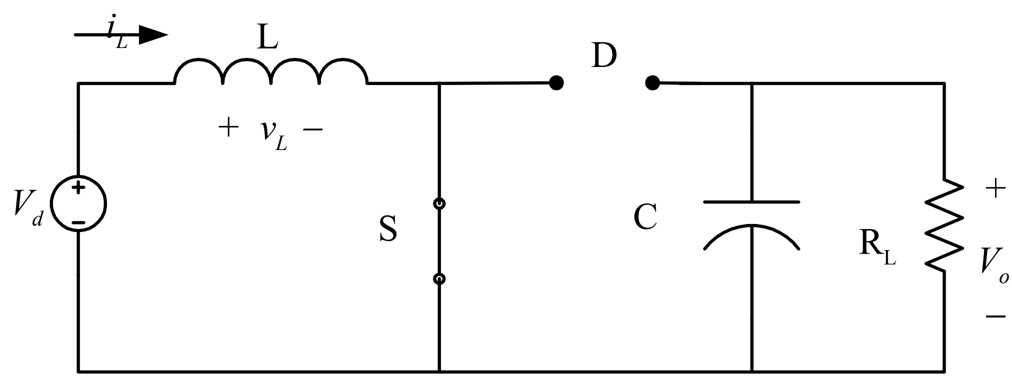
- A buck converter has an input voltage of 50V and output of 25V. The switching frequency is 10KHz. The power output is 125W. (a) Determine the duty cycle, (b) value of L to limit the peak inductor current to 6.25A, (c) value of capacitance to limit the output voltage ripple factor to 0.5%.

- Design a buck converter such that the output voltage is 28V when the input is 48V. The load is 8Ohm. Design the converter such that it will be in continuous current mode. The output voltage ripple must not be more than 0.5%. Specify the frequency and the values of each component. Suggest the power switch also.

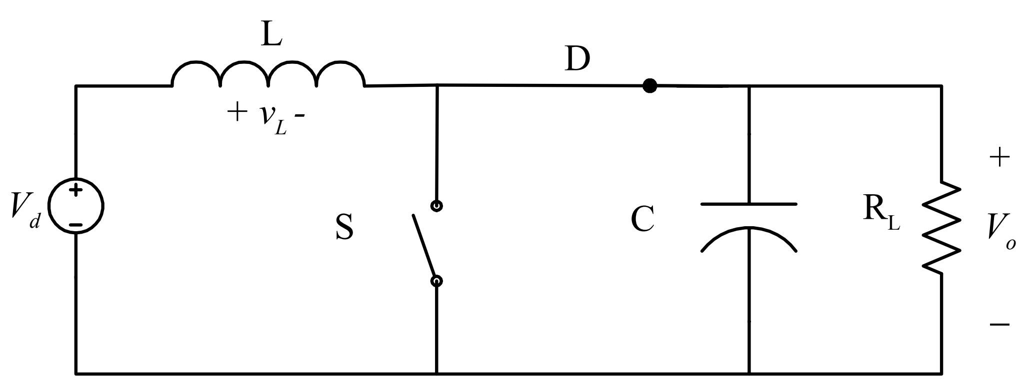
Boost (step-up) converter



CIRCUIT OF BOOST CONVERTER

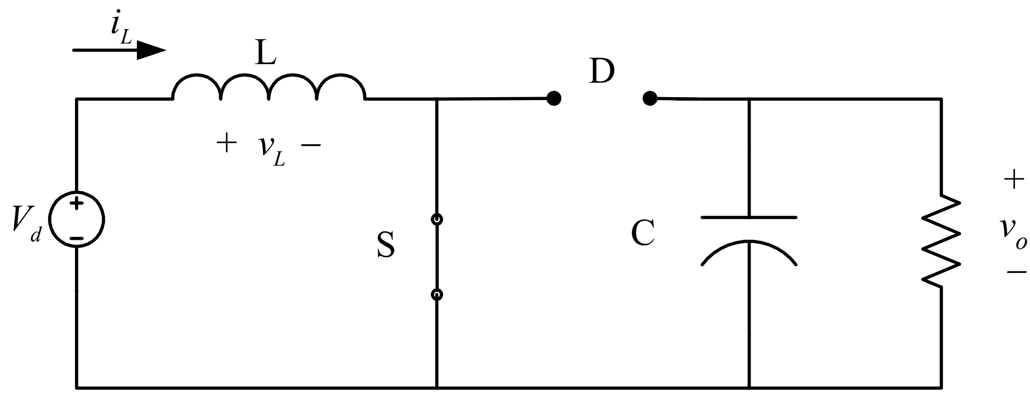


CIRCUIT WHEN SWITCH IS CLOSED



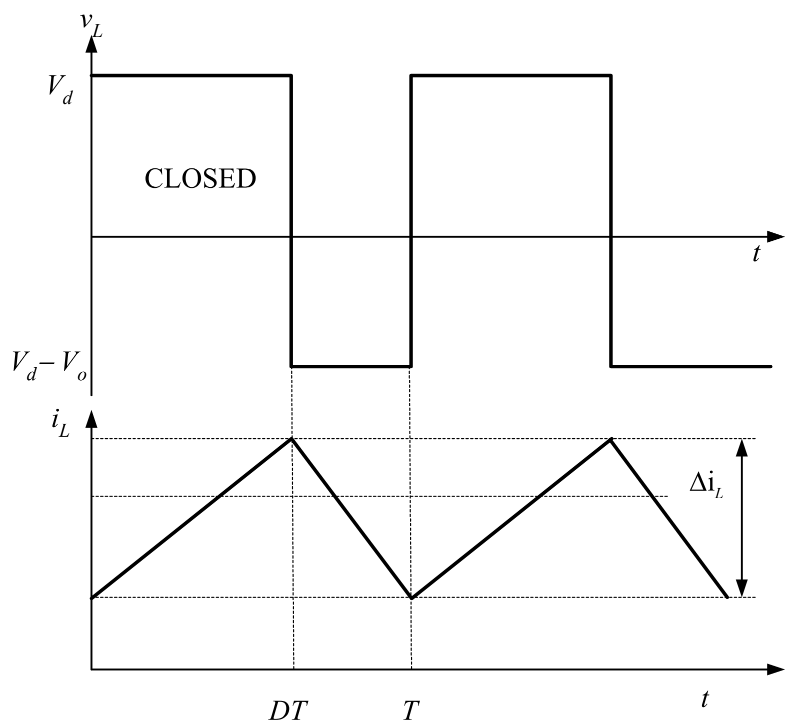
CIRCUIT WHEN SWITCH IS OPENED

Boost analysis:switch closed



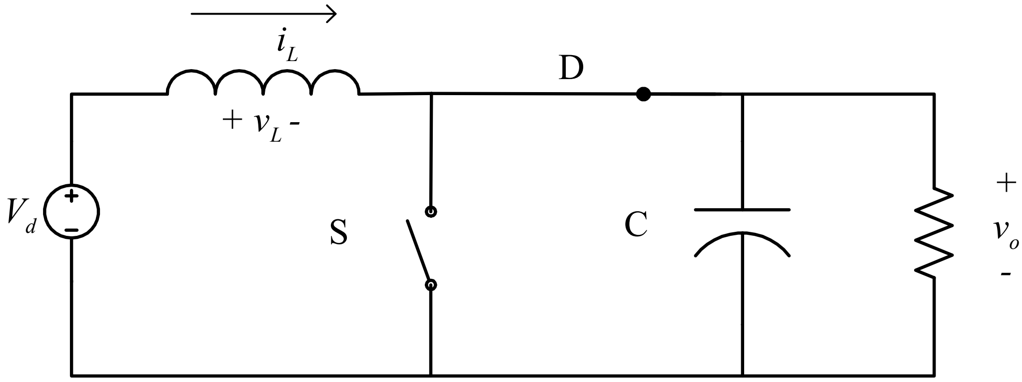
$$v_L = V_d$$
$$= L\frac{di_L}{dt}$$
$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

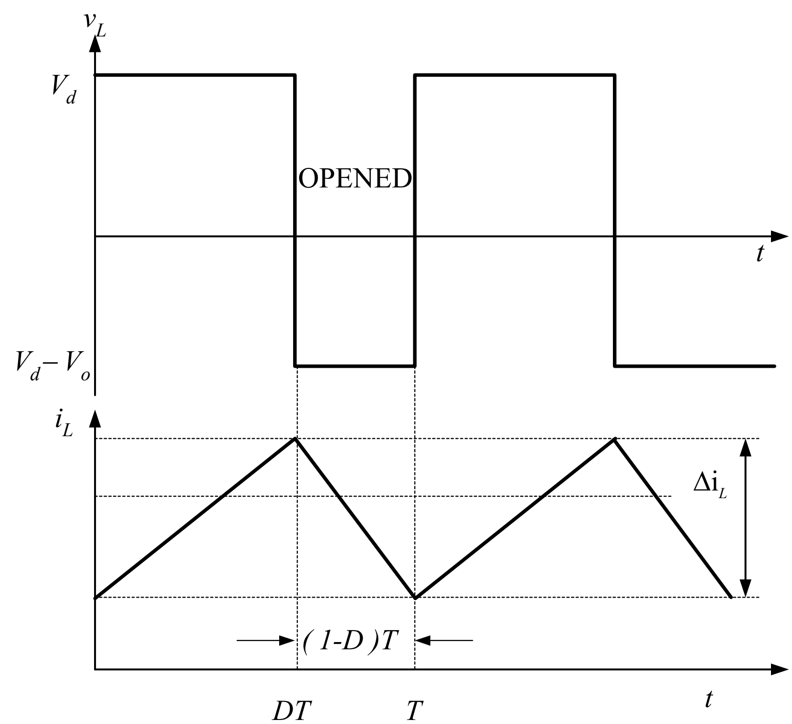
$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT}$$
$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$



$$(\Delta i_L)_{closed} = \frac{V_d DT}{L}$$

Switch opened





$$v_L = V_d - V_o = L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\Rightarrow (\Delta i_L)_{opened} = \frac{(V_d - V_o)(1-DT)}{L}$$

Steady-state operation

$$(\Delta i_L)_{closed} + (\Delta i_L)_{opened} = 0$$

$$\frac{V_d DT}{L} - \frac{(V_d - V_o)(1-D)T}{L} = 0$$

$$\Rightarrow V_o = \frac{V_d}{1-D}$$

- Boost converter produces output voltage that is greater or equal to the input voltage.
- Alternative explanation:
  - when switch is closed, diode is reversed. Thus output is isolated. The input supplies energy to inductor.
  - When switch is opened, the output stage receives energy from the input as well as from the inductor. Hence output is large.
  - Output voltage is maintained constant by virtue of large C.

Average, Maximum, Minimum inductor current

Input power = Output power

$$V_d I_d = \frac{V_o^{\,2}}{R}$$

$$V_d I_L = \frac{\left(\dfrac{V_d}{(1-D)}\right)^2}{R} = \frac{V_d^{\,2}}{(1-D)^2 R}$$

Average inductor current

$$I_L = \frac{V_d}{(1-D)^2 R}$$

Max, min inductor current

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} + \frac{V_d DT}{2L}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L}$$

Continuous Current Mode (CCM)

For continous operation,

$$I_{\min} \geq 0$$

$$\frac{V_d}{(1-D)^2 R} - \frac{V_d DT}{2L} \geq 0$$

$$L_{\min} = \frac{D(1-D)^2 TR}{2}$$

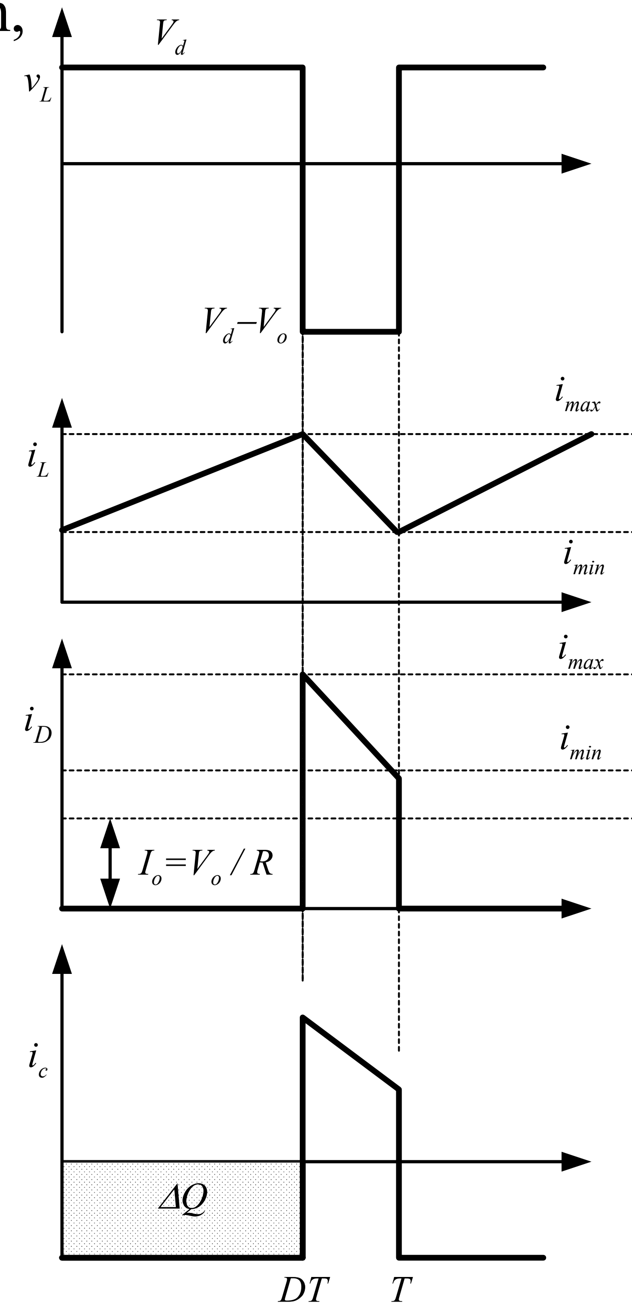
$$= \frac{D(1-D)^2 R}{2f}$$

Ripple factor

$$|\Delta Q| = \left(\frac{V_o}{R}\right) DT = C\Delta V_o$$

$$\Delta V_o = \frac{V_o DT}{RCf} = \frac{V_o D}{RCf}$$

$$r = \frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

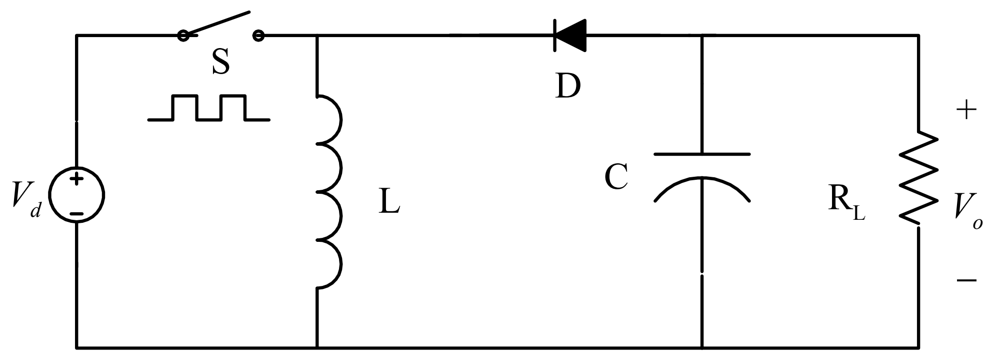


Examples

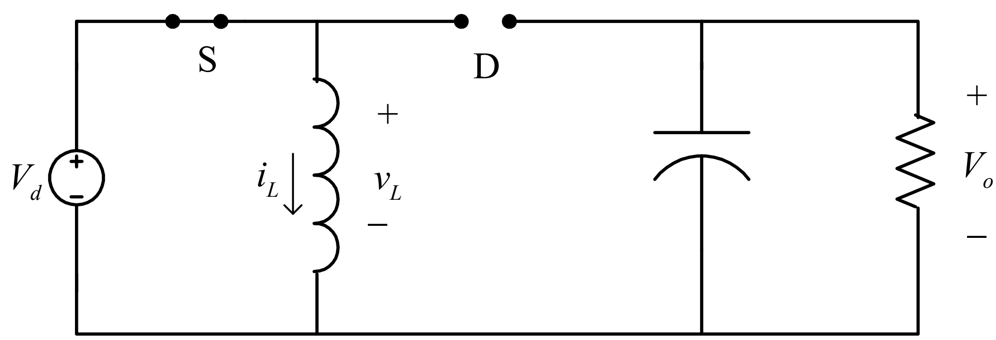
- The boost converter has the following parameters: V_d=20V, D=0.6, R=12.5ohm, L=65uH, C=200uF, f_s=40KHz. Determine (a) output voltage, (b) average, maximum and minimum inductor current, (c) output voltage ripple.

- Design a boost converter to provide an output voltage of 36V from a 24V source. The load is 50W. The voltage ripple factor must be less than 0.5%. Specify the duty cycle ratio, switching frequency, inductor and capacitor size, and power device.

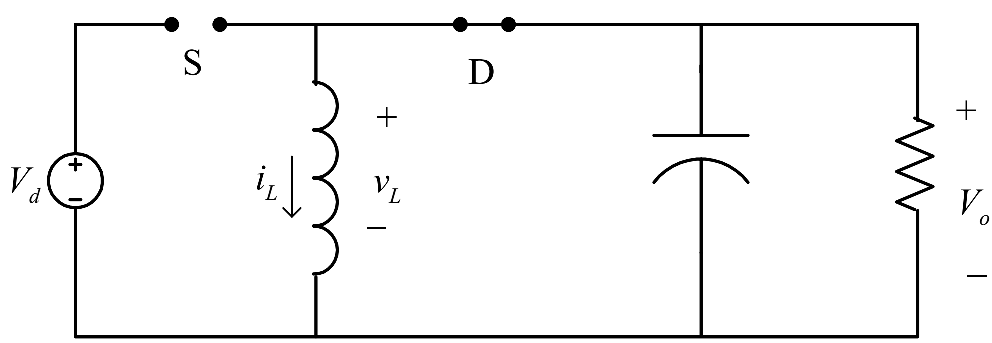
Buck-Boost converter



CIRCUIT OF BUCK-BOOST CONVERTER



CIRCUIT WHEN SWITCH IS CLOSED



CIRCUIT WHEN SWITCH IS OPENED

Buck-boost analysis

Switch closed

$$v_L = Vd = L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_d}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_d}{L}$$

$$(\Delta i_L)_{closed} = \frac{V_d DT}{L}$$

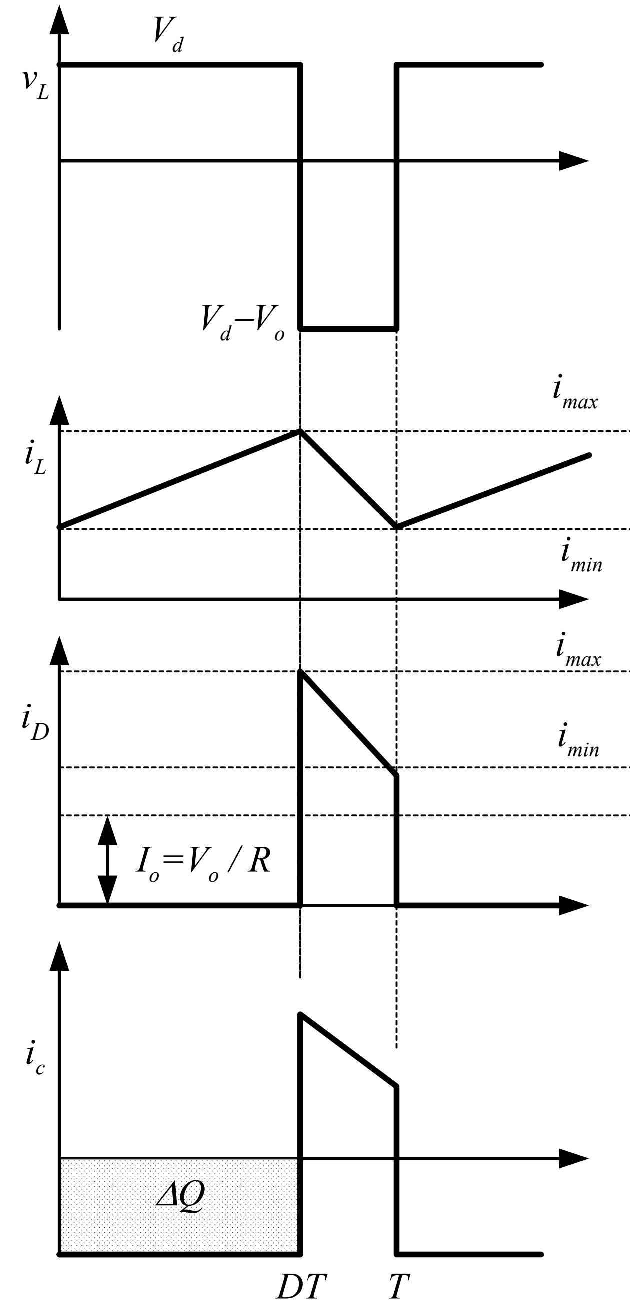
Switch opened

$$v_L = V_o = L\frac{di_L}{dt}$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_o}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_o}{L}$$

$$(\Delta i_L)_{opened} = \frac{V_o(1-D)T}{L}$$



Output voltage

Steady state operation :

$$\frac{V_d DT}{L} + \frac{V_o (1-D)T}{L} = 0$$

$$\Rightarrow \mathrm{V_o} = -V_s \left(\frac{D}{1-D} \right)$$

- NOTE: Output of a buck-boost converter either be higher or lower than the source voltage.
  - If D>0.5, output is higher
  - If D<0.5, output is lower

- Output voltage is always negative
- Note that output is never directly
- connected to load. Energy is stored in inductor when switch is closed and transferred to load when switch is opened.

Average inductor current

Assuming no power loss in the converter, power absorbed by the load must equal power supplied the by source, i.e.

$$P_o = P_s$$

$$\frac{V_o^2}{R} = V_d I_s$$

But average source current is related to average inductor current as :

$$I_s = I_L D$$

$$\Rightarrow \frac{V_o^2}{R} = V_d I_L D$$

Substituting for V_o,

$$\Rightarrow I_L = \frac{V_o^2}{V_d R D} = \frac{P_o}{V_d D} = \frac{V_d D}{R(1-D)^2}$$

L and C values

Max and min inductor current,

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_d D}{R(1-D)^2} + \frac{V_d DT}{2L}$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_d D}{R(1-D)^2} - \frac{V_d DT}{2L}$$

For continuous current,

$$\frac{\mathrm{V_d} D}{R(1-D)^2} + \frac{V_d DT}{2L} = 0$$

$$\Rightarrow L_{\min} = \frac{(1-D)^2 R}{2f}$$

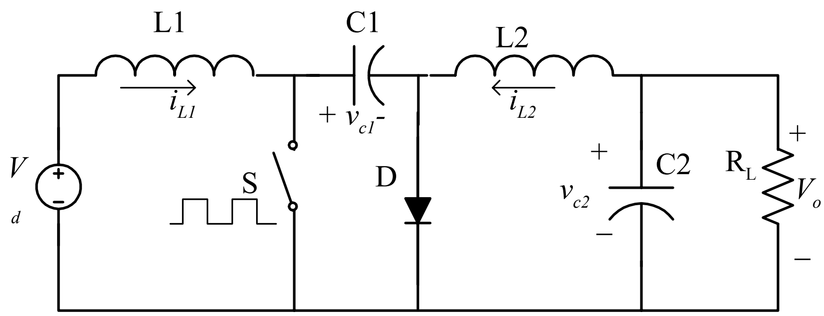
Output voltage ripple,

$$|\Delta \mathrm{Q}| = \left(\frac{V_o}{R}\right) DT = C\Delta V_o$$

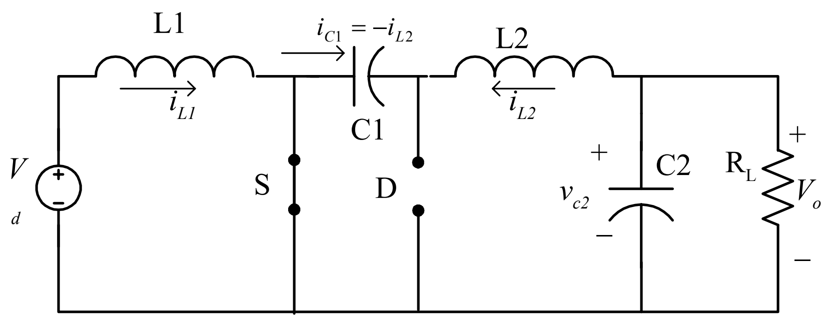
$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$r = \frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

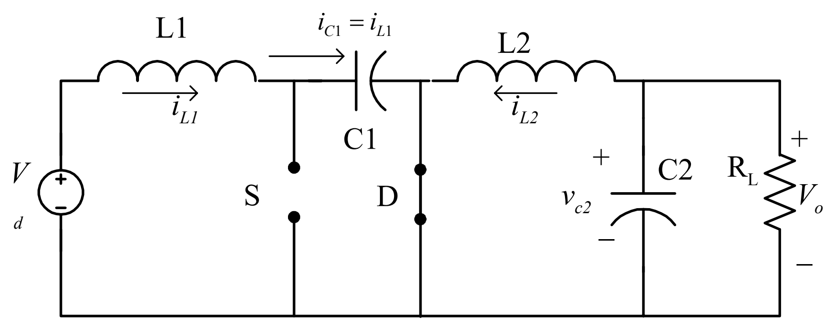
Cuk Converter



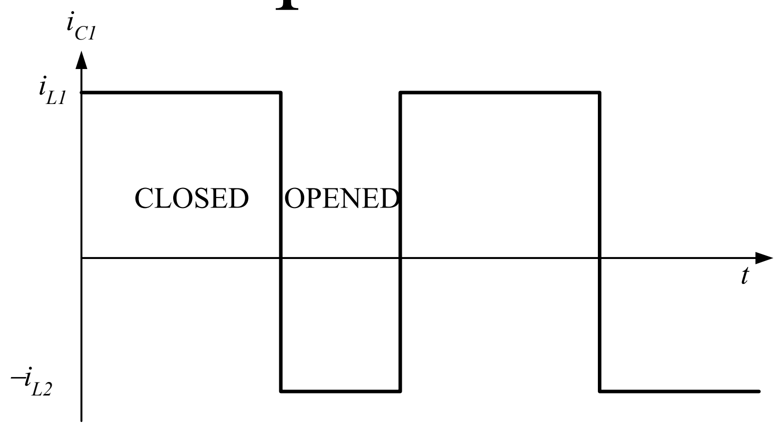
CIRCUIT OF CUK CONVERTER



CIRCUIT WHEN SWITCH IS CLOSED



Cuk analysis: from capacitor current point of view



The average voltage across C1 is computed by KVL,

$$V_{C1} = V_d - V_o$$

When the switch is closed, diode is off and the current in C1 is :

$$(i_{C1})_{closed} = -i_{L2}$$

When the switch is opened, the current in L1 and L2 force the diode on. The current in C1 is :

$$(i_{C1})_{open} = -i_{L1}$$

The power absorbed by the load is equal to the power supplied by the source, i.e.

$$-V_o I_{L2} = V_s I_{L1}$$

For periodic operation, the average current is zero,

$$\left[(i_{C1})_{closed}\right]DT + \left[(i_{C1})_{open}\right](1-D)T = 0$$

Substituting,

$$-I_{L2}DT + I_{L1}(1-D)T = 0$$

$$\frac{I_{L1}}{I_{L2}} = \frac{D}{(1-D)}$$

Cuk analysis

The power absorbed by the load is equal to the power supplied by the source, i.e.

$$-V_o I_{L2} = V_s I_{L1}$$

$$\frac{I_{L1}}{I_{L2}} = \frac{-V_o}{V_s}$$

Combining, output voltage can be written as :

$$\frac{V_o}{V_s} = -\left(\frac{D}{(1-D)}\right)$$

Note that the output stage (L2, C2 adnR) are in the same configuration as the buck converter. Hence,

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8L_2C_2f^2}$$

In time interval DT when switch is closed,

$$v_{L1} = v_d = L_1\frac{di_{L1}}{dt}$$

$$\Rightarrow \frac{\Delta i_{L1}}{DT} = \frac{V_d}{L_1}$$

or

$$\Delta i_{L1} = \frac{V_d DT}{L_1} = \frac{V_d D}{L_1 f}$$

Cuk design parameters

For L2, in time interval DT when switch is closed,

$$0 = +v_{c1} - v_{L2} + V_o$$

$$v_{L2} = v_{c1} + V_o = (V_d - V_o) + V_o = V_d$$

$$v_{L2} = L_2 \frac{di_{L2}}{dt}$$

or

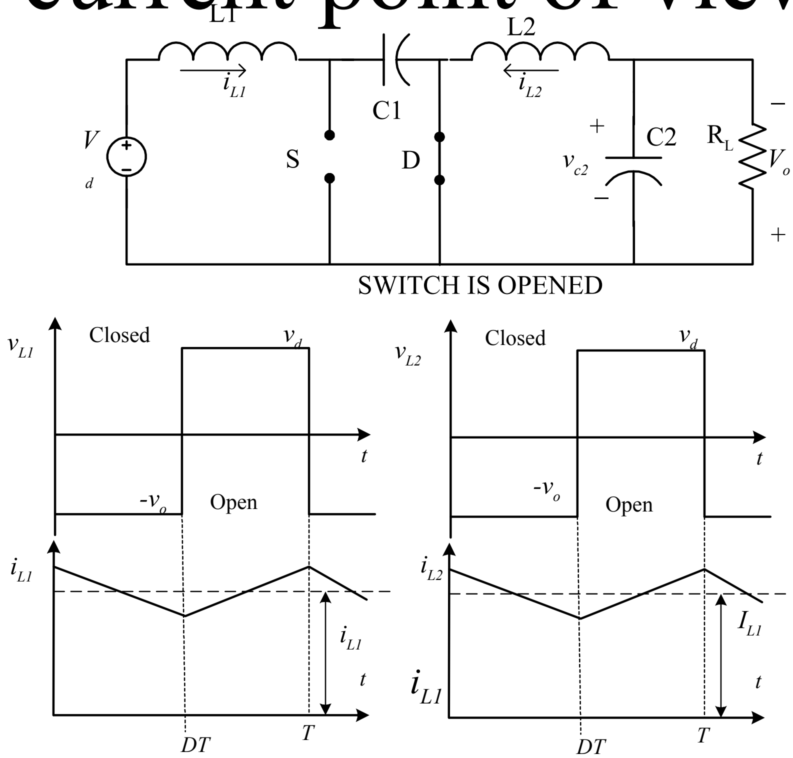
$$\Delta i_{L2} = \frac{V_d DT}{L_2} = \frac{V_d D}{L_2 f}$$

For continuous current operation,

$$L_{1,\min} = \frac{(1-D)^2 R}{2Df}$$

$$L_{2,\min} = \frac{(1-D)R}{2f}$$

Cuk analysis from inductor current point of view



In steady state, it can be assumed that V_{L1} and V_{L2} are zero.

$V_{C1} = V_d + V_o$, (Note the polarity of V_o).

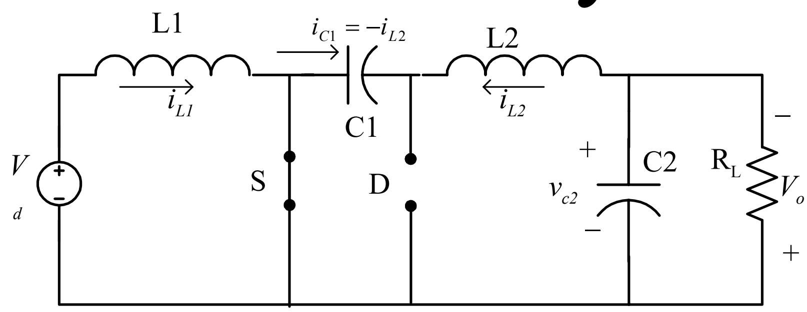
It also can be seen that V_{C1} is larger than V_d and V_0

When the switch is off, i_{L1} and i_{L2} flow through the diode. Capacitor C1 is charged through the diode by energy from Vd and L1. The inductor voltage can be written as:

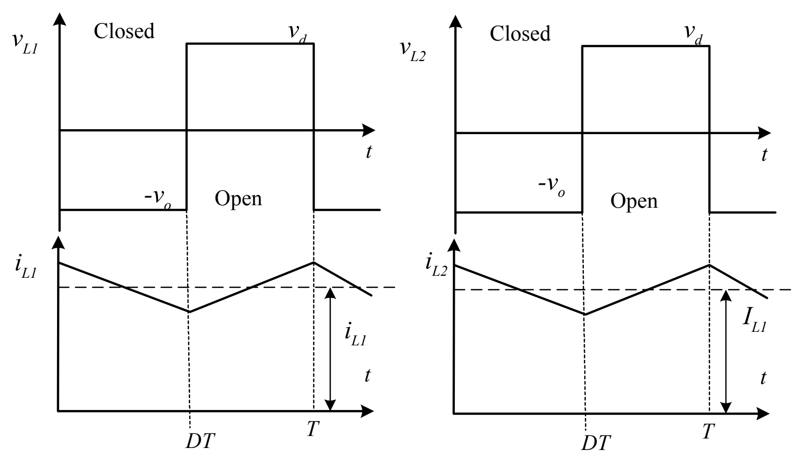
$v_{L1} = V_d + V_{C1}$

which is negative since V_{C1} is larger than V_d. This causes i_{L1} to decrease

Cuk analysis



CIRCUIT WHEN SWITCH IS CLOSED



Similarly on the output side,
$V_o = -V_d$
which which causes i_{L1} to decrease.

When the switch is on, V_{C1} reverse - biased the diode. The inductor current i_{L2} and i_{L2} flow through the switch.

Since $V_{C1} > V_o$ capacitor C1 discharged through the switch, transferring energy to the noutput L2. Therefore i_{L2} increases.

The input feeds energy to L1, causing i_{L1} to increase.

Cuk Analysis

Equating the integral of the voltages across L1 and L2,

$$\text{L1}: V_d DT + (V_d - V_{c1})(1-D) = 0$$

$$\Rightarrow V_{c1} = \frac{1}{1-D} V_d$$

$$\text{L2}: (V_{c1} - V_o)DT + (-V_o)(1-D)T = 0$$

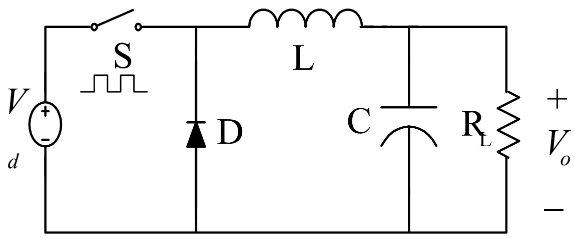
$$\Rightarrow V_{c1} = \frac{1}{D} V_o$$

Combining :

$$\frac{V_o}{V_d} = \frac{D}{1-D}$$

Note the polarity of the output

Converters in CCM: Summary

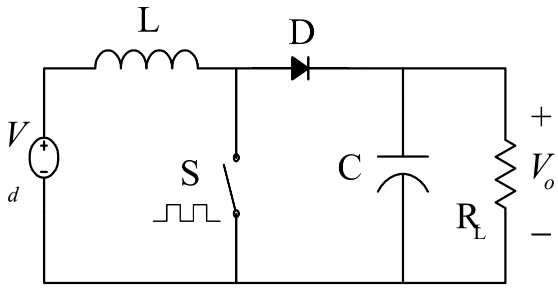


Buck

$$V_o/V_d = D$$

$$\Delta V_o/V_d = \frac{1-D}{8LCf^2}$$

$$L_{\min} = \frac{(1-D)R}{2f}$$

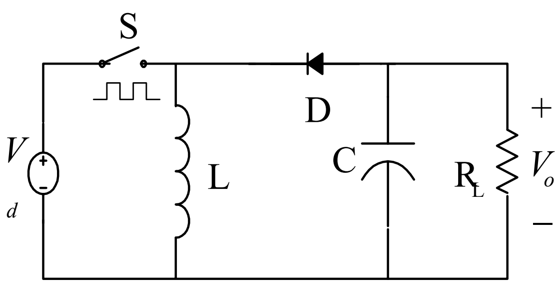


Boost

$$V_o/V_d = \frac{1}{1-D}$$

$$\Delta V_o/V_d = \frac{D}{RCf}$$

$$L_{\min} = \frac{D(1-D)^2 R}{2f}$$

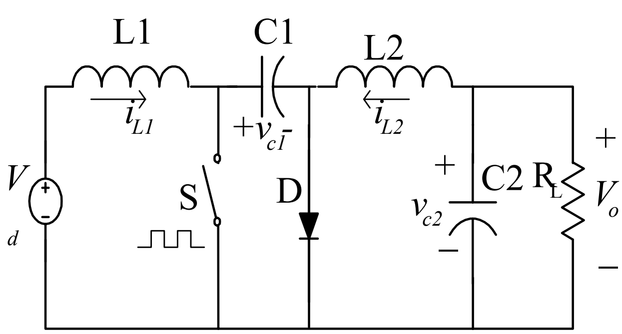


Buck – Boost

$$V_o/V_d = -\frac{D}{1-D}$$

$$\Delta V_o/V_d = \frac{D}{RCf}$$

$$L_{\min} = \frac{(1-D)^2 R}{2f}$$



Cuk

$$V_o/V_d = -\frac{D}{1-D}$$

$$\Delta V_o/V_d = \frac{1-D}{8LCf^2}$$

$$L_1 = \frac{(1-D)^2 R}{2Df}$$

$$L_2 = \frac{(1-D)R}{2f}$$

Buck in discontinuous current mode (DCM)

Average inductor voltage is zero,

$$(V_d - V_o)DT - V_o D_1 T = 0$$

$$\Rightarrow (V_d - V_o)D = V_o D_1$$

$$\frac{V_o}{V_d} = \left(\frac{D}{D + D_1}\right)$$

Average inductor current equals resistor current (because average capacitor current is zero)

$$I_L = I_R = \frac{V_o}{R}$$

From figure,

$$I_L = \frac{1}{T}\left(\frac{1}{2} I_{\max} DT + \frac{1}{2} I_{\max} D_1 T\right)$$

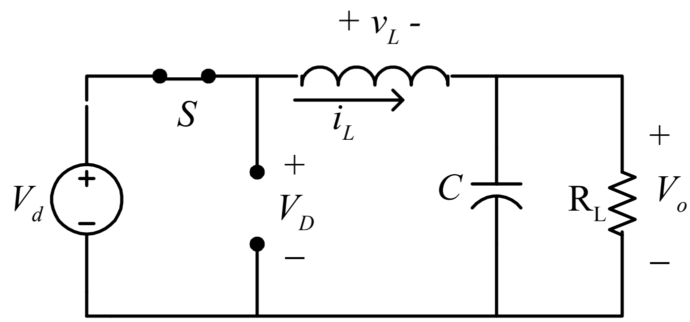
$$= \frac{1}{2} I_{\max}\left(D + D_1 T\right)$$

Voltage across inductor,

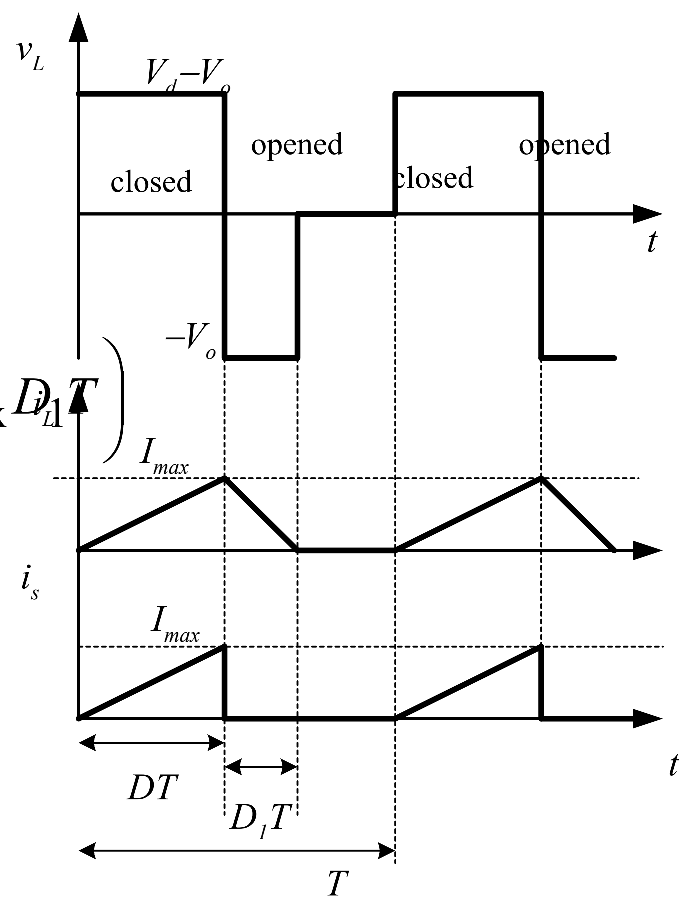
$$v_L = V_d - V_o$$

$$\frac{di_L}{dt} = \frac{V_d - V_o}{L}$$

$$\Rightarrow \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{I_{\max}}{DT}$$



BUCK CONVERTER



Buck in DCM

Solving for I_{max} and using $(V_s - V_o)D$,

$$I_{max} = \Delta i_L = \left(\frac{V_s - V_o}{L}\right)DT = \frac{V_o D_1 T}{L}$$

Substitute,

$$\frac{1}{2} I_{max}\left(D + D_1\right) = \frac{1}{2}\left(\frac{V_o D_1 T}{L}\right)\left(D + D_1\right) = \frac{V_o}{R}$$

Which gives,

$$D_1{}^2 + DD_1 - \frac{2L}{RT} = 0$$

Solving for D_1,

$$D_1 = \frac{-D\sqrt{D^2 + \frac{8L}{RT}}}{2}$$

Hence,

$$V_o = V_d\left(\frac{D}{D + D_1}\right) = V_d\left(\frac{2D}{D + \sqrt{D^2 + \frac{8L}{RT}}}\right)$$

Example

For the buck converter,
$V_d = 24V, L = 200uH, R = 20\Omega, C = 100uF, f = 10KHz, D = 0.4$
a) Show that the inductor current is discontinuous :
b) Determine the output voltage, V_o

For discontinous current, $D_1 < 1 - \text{D}$
D_1 can be calculated by :

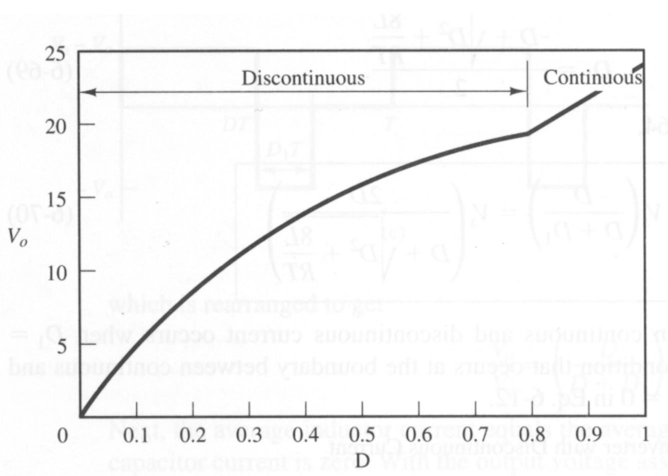
$$D_1 = \frac{-D\sqrt{D^2 + \frac{8L}{RT}}}{2}$$

$$= \frac{1}{2}\left(-0.4 + \sqrt{0.4^2 + \frac{8(200)(10^{-6})(10K)}{20}}\right) = 0.29$$

Since $D_1 < (1 - \text{D})$, i.e. $0.29 < 0.64$, circuit in DCM

$$V_o = V_d\left(\frac{D}{D + D_1}\right) = 13.97V.$$

Figure below shows the relationship between the output voltage and duty ratio for the parameters of this example.



Boost Converter in DCM

Average inductor voltage is zero,

$$V_d DT + (V_d - V_o) D_1 T = 0$$

$$\Rightarrow (V_d - V_o) D = V_o D_1$$

$$\frac{V_o}{V_d} = \left(\frac{D + D_1}{D}\right)$$

Average diode current is :

$$I_D = \frac{1}{T}\left(\frac{1}{2} I_{\max} D_1 T\right) = \frac{1}{2} I_{\max} D_1$$

is the same as the change in inductor current when the switch is closed,

$$I_{\max} = \Delta i_L = \frac{V_d DT}{L}$$

$$I_D = \frac{1}{2}\left(\frac{V_d DT}{L}\right) D_1 = \frac{V_o}{R}$$

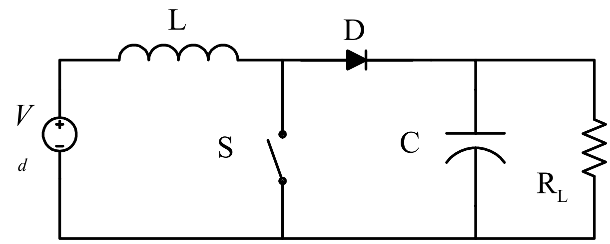
Solving for D_1,

$$D_1 = \left(\frac{V_o}{V_d}\right)\left(\frac{2L}{RDT}\right)$$

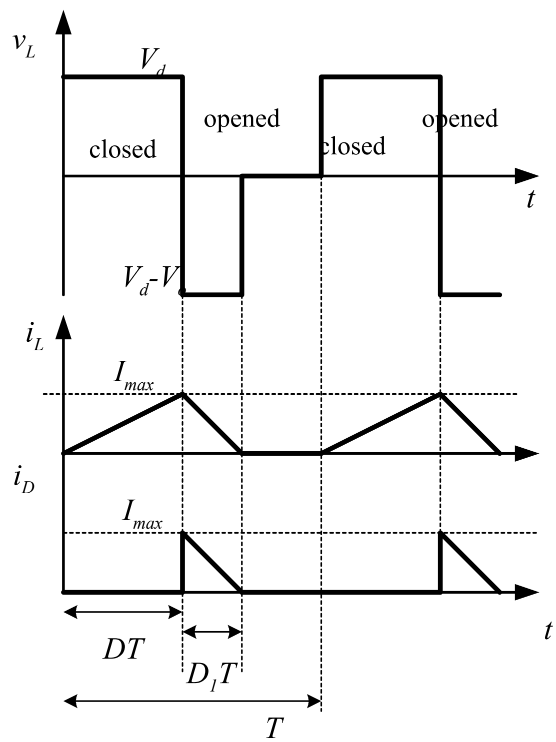
Substituting,

$$\left(\frac{V_o}{V_d}\right)^2 - \left(\frac{V_o}{V_d}\right) - \left(\frac{D^2 RT}{2L}\right) = 0$$

$$\frac{V_o}{V_d} = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2D^2 RT}{L}}\right)$$

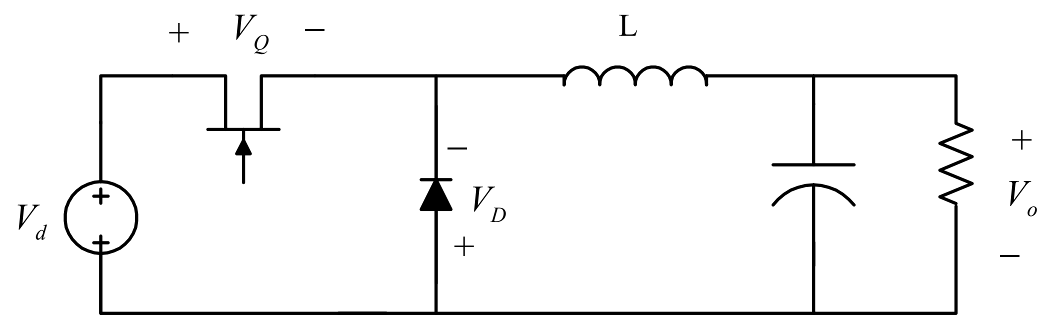


BOOST CONVERTER



Non-ideal effects: switch/diode voltage drop

Example : Buck converter



During switch closed (on),

$v_L = V_d - V_o - V_Q$

where V_Q is the voltage across the conducting switch

During switch open (off),

$v_L = -V_d - V_D$

where V_D is the voltage across the diode.

The average voltage across the inductor is zero for the switching period,

$V_L = (V_d - V_o - V_Q)D + (-V_d - V_D)(1-D) = 0,$

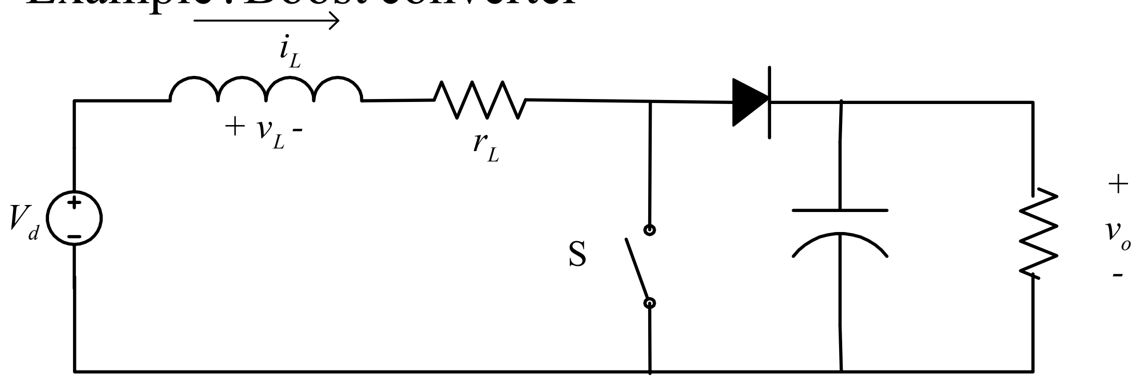
Solving,

$$V_o = V_d D - V_Q D - V_D(1-D)$$

Which is less than $V_o = V_d D$ for the ideal case.

Inductor (winding) resistance

Example : Boost converter



Power absorbed by the load and the inductor resitance (rs), must equal power supplied by the source, i.e.

$$P_s = P_o + P_{rL}$$

$$V_d I_L = V_o I_D + I_L{}^2 r_L$$

But, the average (DC) diode current,

$$I_D = I_L(1 - D)$$

Substituting,

$$V_d I_L = V_o I_L(1 - D) + I_L{}^2 r_L$$

Which becomes,

$$V_d = V_o(1 - D) + I_L r_L$$

Inductor resistance

But,

$$I_d = \frac{I_D}{(1-D)} = \frac{V_o/R}{(1-D)}$$

Hence,

$$V_d = \frac{V_o r_L}{R(1-D)} + V_o(1-D)$$

Solving,

$$V_o = \left(\frac{V_d}{(1-D)}\right)\left(\frac{1}{1+\dfrac{r_L}{R(1-D)^2}}\right)$$

The output equation is similar for ideal boost converter but includes a correction factor to account for inductor resistance

Efficiency :

$$\eta = \frac{P_o}{P_o + P_{loss}} = \frac{V_o^2/R}{V_o^2/R + I_L^2 r_L}$$

Substituting for I_L,

$$\eta = \frac{V_o^2/R}{V_o^2/R + \left(\dfrac{V_o^2/R}{(1-D)}\right) r_L} = \frac{1}{1+\dfrac{r_L}{R(1-D)^2}}$$

As the duty ratio increases, the efficiency of boost converter decreases.

Other non-idealities

- Capacitor's Equivalent Series Resistor (ESR)
  - Producing ripple greater than ideal capacitor
  - Output C must be chosen on the basis of ESR and not only capacitance value.

- Switching losses

Switch-mode power supply (SMPS)

- Advantages over linear power
  -Efficient (70-95%)
  -Weight and size reduction

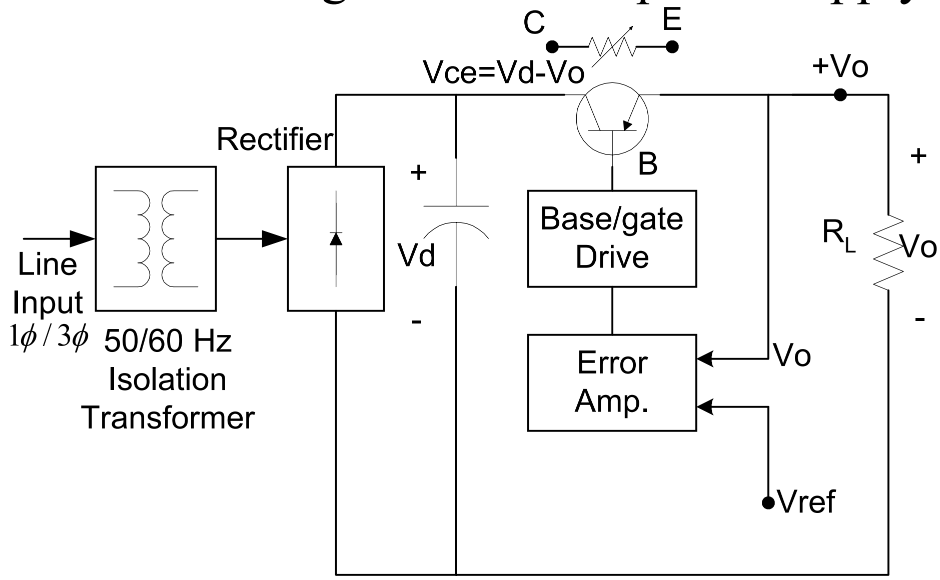
- Disadvantages
  -Complex design
  -EMI problems

- However above certain ratings, SMPS is the only feasible choice

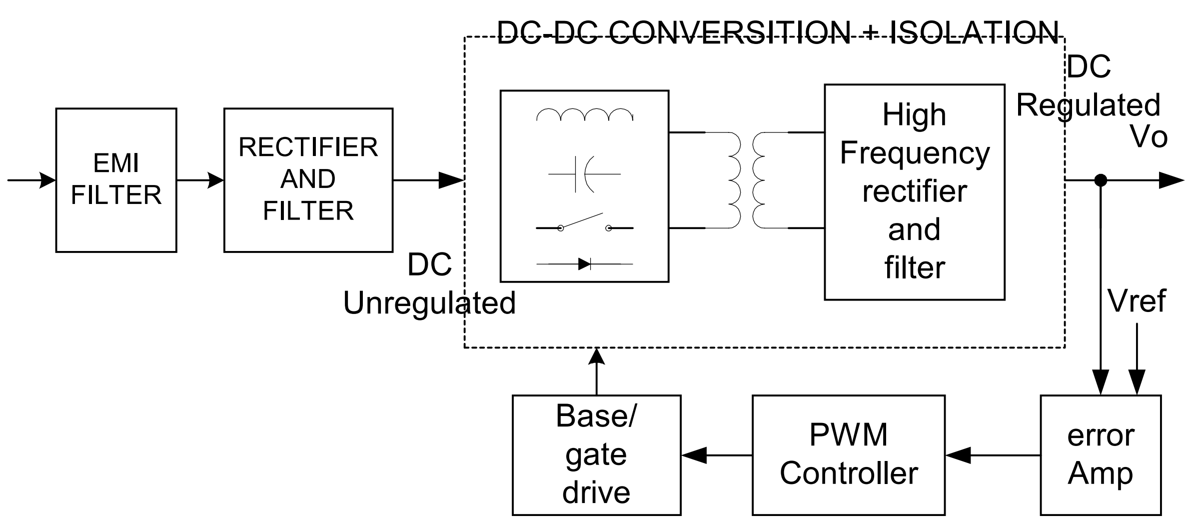
- Types of SMPS
  -Flyback
  -forward
  -Push-pull
  -Bridge (half and full)

Linear and switched mode power supplies block diagram

Basic Block diagram of linear power supply



Basic Block diagram of SMPS



High frequency transformer

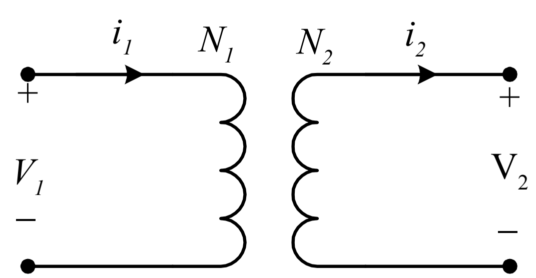
Basic function :

i) Input - output electrical isolation

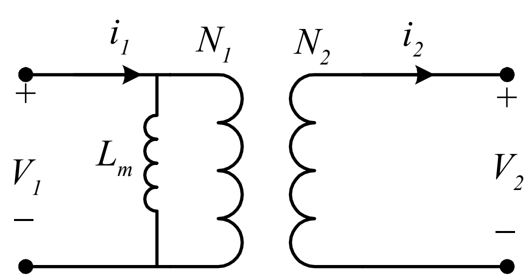
ii) step up/down time - varying voltage
Basic input - output relationship

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}; \qquad \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

Models:

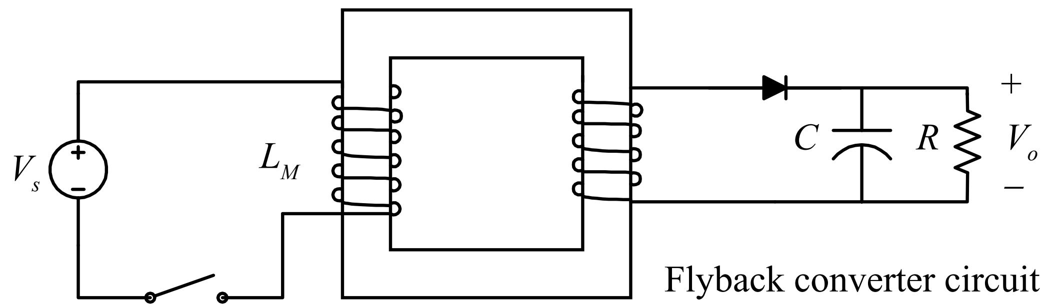


Ideal model

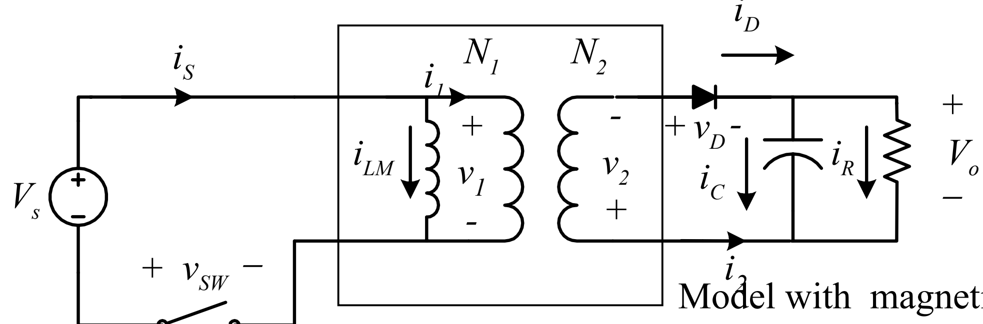


Model used for
most PE application

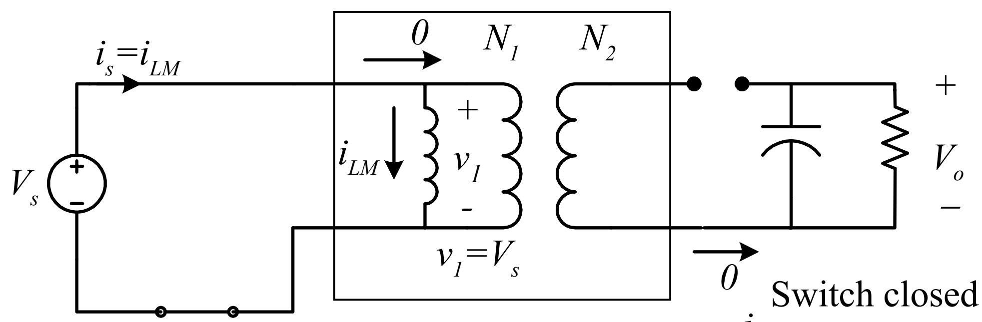
Flyback Converter



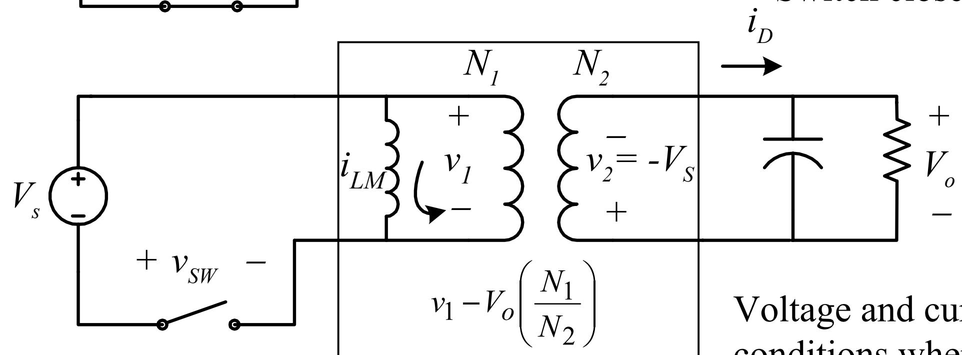
Flyback converter circuit



Model with magnetising inductance



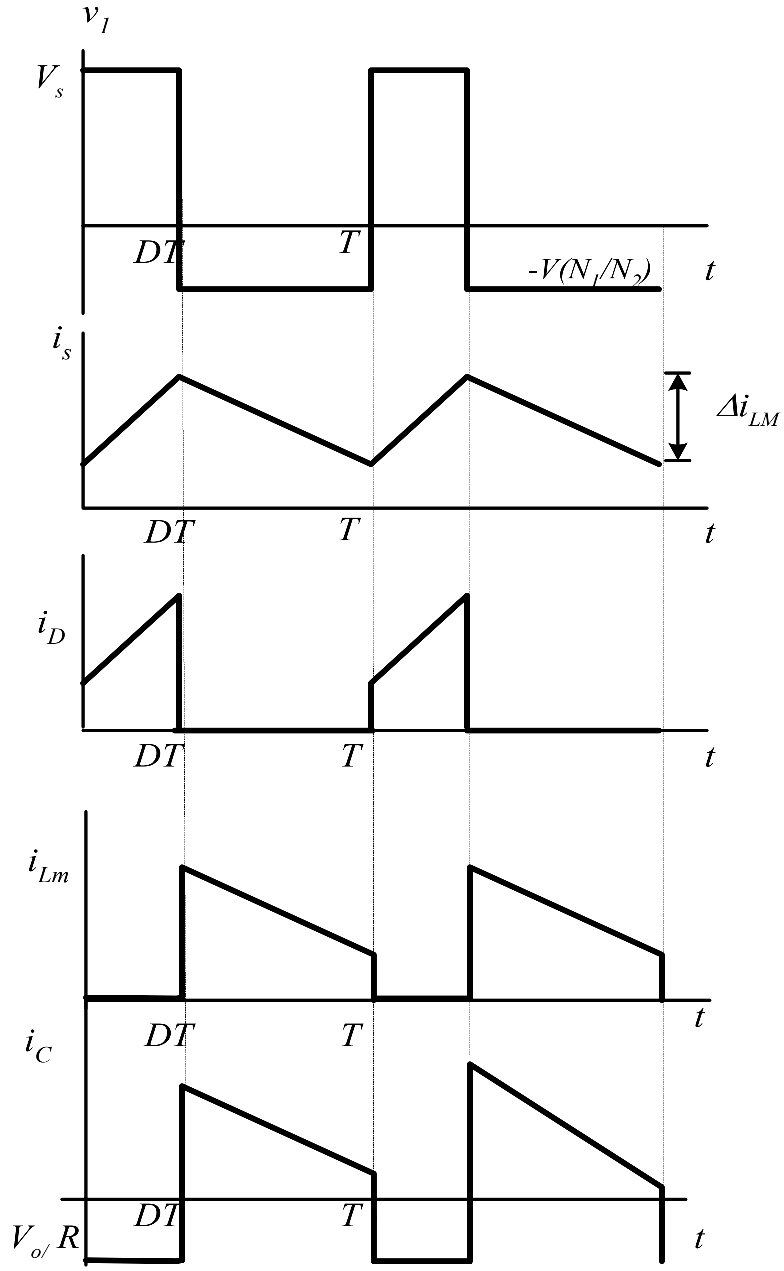
Switch closed



$$V_{sw} = V_s + V_O\left(\frac{N_1}{N_2}\right)$$

Voltage and current conditions when switch opened

Flyback waveforms



Analysis: switched closed

$$v_1 = V_d = L_m \frac{di_{Lm}}{dt}$$

$$\frac{diL_m}{dt} = \frac{\Delta iL_m}{dt} = \frac{\Delta iL_m}{DT} = \frac{V_d}{L_m}$$

$$\Rightarrow \left(\Delta i_{L_m}\right)_{closed} = \frac{V_d DT}{L_m}$$

On the load side of the transformer,

$$v_2 = v_1\left(\frac{N_2}{N_1}\right) = V_d\left(\frac{N_2}{N_1}\right)$$

$$v_D = -V_o - V_d\left(\frac{N_2}{N_1}\right) < 0$$

Therefore,

$$i_1 = 0$$
$$i_2 = 0$$

Analysis: switch opened

$$v_1 = -V_0\left(\frac{N_1}{N_2}\right); \qquad v_2 = -V_0$$

$$\Rightarrow v_1 = v_2\left(\frac{N_1}{N_2}\right) = -V_0\left(\frac{N_1}{N_2}\right)$$

$$L_m \frac{di_{Lm}}{dt} = v_1 = -V_0\left(\frac{N_1}{N_2}\right)$$

$$\frac{di_{Lm}}{dt} = \frac{\Delta i_{Lm}}{dt} = \frac{\Delta i_{Lm}}{(1-D)T} = \frac{-V_0}{L_m}\frac{N_1}{N_2}$$

$$\Rightarrow \left(\Delta i_{Lm}\right)_{open} = -\frac{V_0(1-D)T}{L_m}\left(\frac{N_1}{N_2}\right)$$

For steady-state operation,

$$\left(\Delta i_{L_m}\right)_{closed} + \left(\Delta i_{L_m}\right)_{opened} = 0$$

$$\Rightarrow \frac{V_d DT}{L_m} + \frac{V_0(1-D)T}{L_m}\left(\frac{N_1}{N_2}\right) = 0$$

$$\Rightarrow V_0 = V_d \frac{D}{(1-D)}\left(\frac{N_1}{N_2}\right)$$

Output voltage

- Input output relationship is similar to buck-boost converter.
- Output can be greater of less than input,depending upon D.
- Additional term, i.e. transformer ratio is present.

Average inductor current

$$P_s = P_0$$
$$V_d I_s = \frac{{V_0}^2}{R}$$

I_s is related to I_{L_m} as:

$$I_s = \frac{I_{L_m} DT}{T} = \left(I_{L_m}\right)D$$

Substitute and solving for I_{L_m}

$$V_d\left(I_{L_m}\right)D = \frac{V_0^2}{R}$$
$$\Rightarrow I_{L_m} = \frac{{V_0}^2}{V_d DR}$$

The average inductor current is also written as:

$$I_{L_m} = \frac{V_d D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 = \frac{V_0}{(1-D)R}\left(\frac{N_2}{N_1}\right)$$

Max, Min inductor current, L_{min}, C values

$$I_{L_m,\max} = I_{L_m} + \frac{\Delta i_{L_m}}{2} = \frac{V_d D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 + \frac{V_d DT}{2L_m}$$

$$I_{L_m,\min} = I_{L_m} - \frac{\Delta i_{L_m}}{2} = \frac{V_d D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 - \frac{V_d DT}{2L_m}$$

For continuos operation, $I_{L_m,\min} = 0$

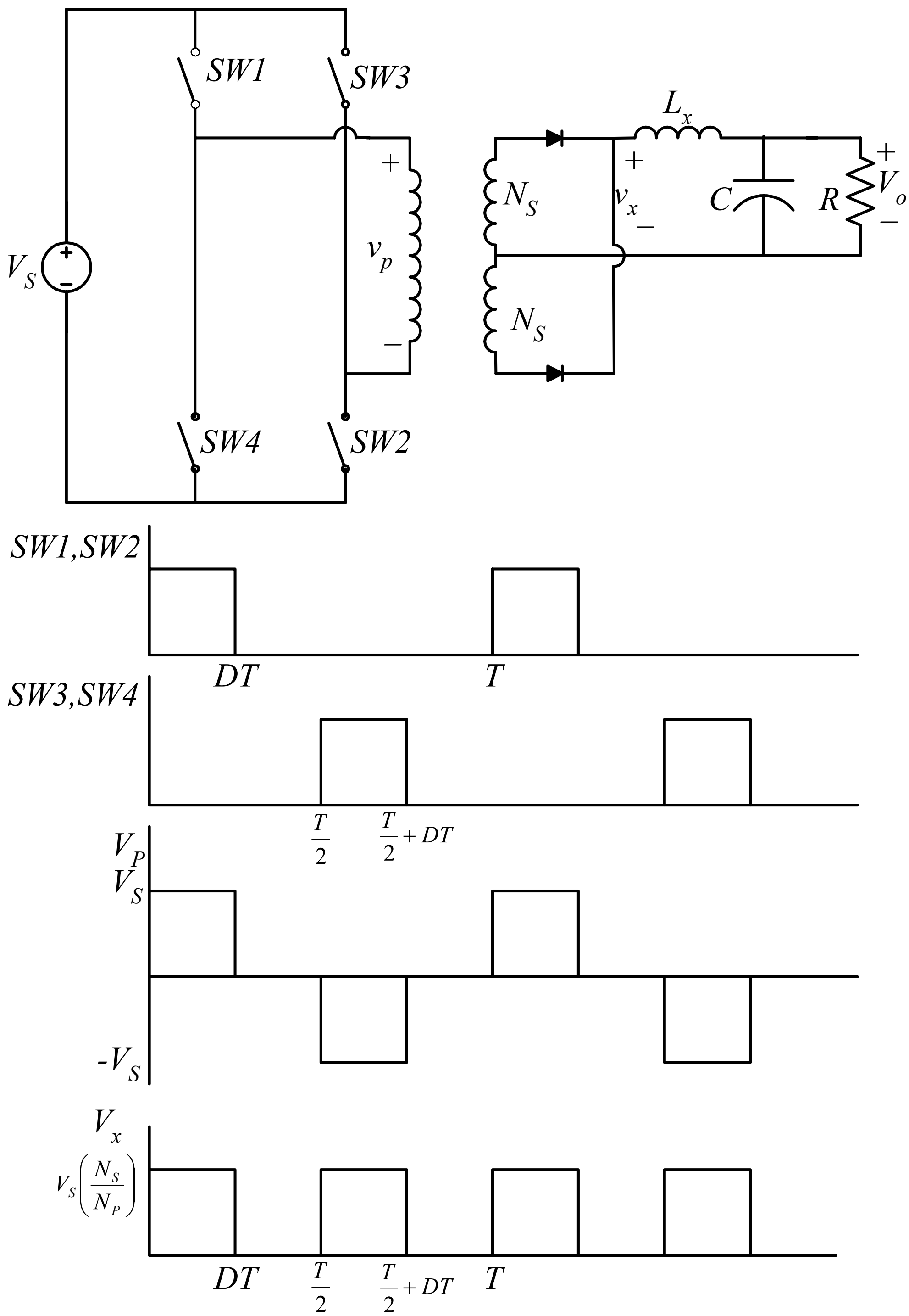
$$\frac{V_d D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 = \frac{V_d DT}{2L_m} = \frac{V_d D}{2L_m f}$$

$$(L_m)_{\min} = \frac{V_d (1-D)^2 R}{2f}\left(\frac{N_1}{N_2}\right)^2$$

The ripple calculation is similar to boost converter,

$$r = \frac{\Delta V_0}{V_0} = \frac{D}{RCf}$$

Full-bridge converter



Full bridge: basic operation

- Switch “pair”: [S1 & S2];[S3 & S4].
- Each switch pair turn on at a time as shown. The other pair is off.
- “AC voltage” is developed across the primary. Then transferred to secondary via high frequency transformers.
- On secondary side, diode pair is “high frequency full wave rectification”.
- The choke (L) and © acts like the “buck converter” circuit.
- Output Voltage $V_o = 2V_s \left(\frac{N_s}{N_p} \right) \cdot D$

Control of DC-DC Converter