حل تمرین دست نویس

نظریه زبان ها و ماشین ها

تهیه کننده و گردآورنده:

BORNA66

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CH # 1.2 : EXERCISES

③ PT. $(w^R)^R = w \quad \forall w \in \Sigma^*$

(Induction)

$(uv)^R = v^R u^R$

(n+1) length : $v = wa$

$(uwa)^R = (wa)^R u^R$

$= a w^R u^R$

$\therefore$ true $\forall w \in \Sigma^*$

④ $L = \{ab, aa, baa\}$
which of the following strings are in L^*, L^4?

abaabaabaa	L^4, L^*
aaaabaaaa	L^4, L^*
baaaaabaaaab	
baaaaabaa	L^4, L^*

⑤ let $\Sigma = \{a, b\}$ Use set notation to describe $\bar{L}$.

$L = \{aa, bb\}$

$\bar{L} = U - \{aa, bb\}$

$\bar{L} = \{\lambda, a, b, ab, ba\} \cup \{w : |w| > 2, w \in \Sigma^*\}$

⑥ Let L be any language on a non-empty alphabet. Show that $L, \bar{L}$ cannot be both finite.

Case (i) L is finite

→ we know U is infinite

→ $\bar{L} = U - L$

= infinite lang − finite lang

= infinite lang.

Case (ii) L is infinite

U is infinite

$\bar{L} = U - L$

= finite

From above, in any case, both cannot be finite

(7) Are there any languages for which $\overline{L^*} = (\overline{L})^*$?

$\lambda \in L^*$

$\Rightarrow \lambda \notin \overline{L^*}$

but $(\overline{L})^*$ contains λ

$\therefore$ No language satisfies $\overline{L^*} = (\overline{L})^*$

(8) PT. $(L_1L_2)^R = L_2^R L_1^R \quad \forall \; L_1 L_2$

let $u \in L_1$, $v \in L_2$

$L_1 L_2 \Rightarrow uv$

$(L_1L_2)^R \Rightarrow (uv)^R$

$= v^R u^R = L_2^R L_1^R \quad \forall \; u, v.$

$\therefore (L_1L_2)^R = L_2^R L_1^R \quad \forall \; L_1, L_2$

(9) Show that $(L^*)^* = L^* \; \forall \; L.$

let $\Sigma = \{a, b\}$

$L^* \Rightarrow \Sigma^* \Rightarrow \{a, b\}^*$

$(L^*)^* \Rightarrow (\{a, b\}^*)^* = \{a, b\}^*$

$= L^* \quad \forall \; \Sigma$

(10) (a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R \quad \forall \; L_1, L_2$

$(L_1 \cup L_2)^R \Rightarrow$ if $u \in L_1$ $v \in L_2$

$uv \in L_1 \cup L_2$,

$(L_1 \cup L_2)^R = (uv)^R = v^R u^R$

$\neq L_1^R \cup L_1^R$

(EXERCISES)

10. (b)

$(L^R)^* = (L^*)^R \quad \forall L$

let $uv \in L$

$L^R = (uv)^R$

$= v^R u^R$

$(L^R)^* = (v^R u^R)^*$

$L^* = (uv)^*$

$(L^*)^R = [(uv)^*]^R$

$\therefore (L^R)^* \neq (L^*)^R$

(11) Find the Grammars that generate the sets of following for $\Sigma = \{a, b\}$

(a) all strings with exactly one a.

P:

$S \to AaA$
$A \to bA / \lambda$

$G = (\{A, S\}, \{a, b\}, S, P)$

(b) all strings with atleast one a.

P:

$S \to AaA$
$A \to aA / bA / \lambda$

$G = (\{A, S\}, \Sigma, S, P)$

(c) all strings with no more than 3 a's.

P: ~~$S \to AaAaAaA$~~ 0,1,2,3 a's
~~$A \to bA / \lambda$~~

$G = (\{A, S\}, \Sigma, S, P)$

$S \to AaA / AaAaA / AaAaAaA$
$A \to bA / \lambda$

$S \to AaA / AaAaA / AaAaAaA / \lambda$
$A \to bA / \lambda$

(d) All strings with atleast 3 a's.

P: $S \to AaAaAaA$
$A \to aA / bA / \lambda$

$G = (\{A, S\}, \Sigma, S, P)$

test:

aaa : $S \to AaAaAaA \to aaa$

baababa: $S \to AaAaAaA \to baAaAaA \to baaAaAaA \to baabaAaA \to$ baababa ✓

(12) $S \to aA$
$A \to bS$
$S \to \lambda$

$ab, abab \ldots$

$L = \{(ab)^n : n \geq 0\}$

(13) What language does the Grammar with these productions generate?

$S \to Aa$
$A \to B$
$B \to Aa$

$S \to Aa \to Ba \to Aaa$

$L = \emptyset$: no terminal symbol to generate strings.

(14) $\Sigma = \{a, b\}$. For each of below languages, find a grammar that generates it

(a) $L_1 = \{a^n b^m : n \geq 0, m > n\}$

P_1:
$S \to Ab$
$A \to aAb \,/\, \lambda \,/\, Ab$

$G : (\{A, S\}, \{a, b\}, S, P_1)$

$S \to aSb | B$
$B \to bB | b$

test: b : $S \to Ab \to b$ ✓

ab : $S \to Ab$ ×

$abb \to Ab \to aAbb \to abb$ ✓

$bb \to S \to Ab \to bb$

(b) $L_2 = \{a^n b^{2n} : n \geq 0\}$

P_2:
$S \to aSbb \,/\, \lambda$

$G : (\{S\}, \Sigma, S, P_2)$

tests:
λ : $S \to \lambda$
abb : $S \to aSbb \to abb$ ✓

aab : $S \to aSbb$ ×

(c) $L_3 = \{a^{n+2} b^n : n \geq 1\}$

$S \to aaA$
$A \to aAb \,/\, \lambda$

test. $n=1$: $a^3 b^1$: $S \to aaA \to aaaAb$
$\to aaab$

14.

(c)

$L_4 = \{a^n b^{n-3} : n \geq 3\}$

$n-3 = m$

$n = m+3$

$n \geq 3$

$m \geq 0$

$\Rightarrow L_3 = \{a^{m+3} b^m : m \geq 0\}$

P_3:

$S \to aaaA$

$A \to aSb/\lambda$

$\therefore G = (\{A, S\}, \Sigma, S, P_3)$

d) $L_8 = L_1 L_2$

$L_5 = \{a^n b^m a^n b^{2n} : n \geq 0, m > n\}$

$= \{a^n b^m a^k b^{2k} : n, k \geq 0, m > n\}$

$S \to S_1 S_2$

$S \to AB$

$A \to Cb$

$C \to aCb/\lambda/Cb$

$B \to aBbb/\lambda$

$\Rightarrow$

$S \to AbB$

$A \to aAb/\lambda/Ab$

$B \to aBbb/\lambda$

test: abbabb: $S \to AbB \to aAbbB \to abbB \to$ abbabb ✓

b: $S \to AbB \to bB \to b$ ✓

bb: ✗

$S \to S_1/S_2$

(e) $L_1 \cup L_2$:

$S \to Ab/B$

$A \to aAb/\lambda$

$B \to aBbb/\lambda$

(g) L_1^3: $\{a^nb^ma^nb^ma^nb^m : n \geq 0, m > n\}$

$S \to AbAbAb$ $\quad S \to SS_1S_1$

$A \to aAb / Ab / \lambda$

Test: $n=0 : m=1$ $\quad bbb$

$S \to AbAbAb \to bbb$

$n=0 : m=2$ $\quad bbbbbb$

$S \to AbAbAb \to bbbbbb$

reject: $ababaaba$

$S \to AbAbAb$

$\to aAbbAbAb$ ×

(h) L_1^*: $\quad L_1 = \{a^nb^m : n \geq 0, m > n\}$

$S \to SA / \lambda$ $\quad S \to SS_1 | \lambda$

$A \to aAb / Ab / \lambda$

test: λ : $S \to \lambda$

$abbaabbb$: $S \to SA \to SaAb \to SaaAbb \to Saabbb \to$

$SAaabbb \to aAbaabbb \to abbaabbb$ ✓

reject: aba : $S \to SA \to SaAb$ ×

(i) $L_1 - \overline{L_4}$: $\quad L_4 = \{a^{n+3}b^n : n \geq 0\}$

$L_1 = \{a^nb^m : n \geq 0, m > n\}$

$L_1 - \overline{L_4} = L_1 - (U - L_4)$

$= L_1 - U + L_4$

$= L_1 + L_4 - U \quad = \emptyset$

(15) Find the grammars for the following on $\Sigma = \{a\}$

(a) $L = \{ w : |w| \bmod 3 = 0 \}$

$S \to aaaS / \lambda$

(b) $L = \{ w : |w| \bmod 3 > 0 \}$

1, 2, 4, 5, 7, 8, 10, 11

$S \to A / B$

$A \to aaaA / a.$

$B \to aaaB / aa$

$S \to aaaS / a / aa$

Test:

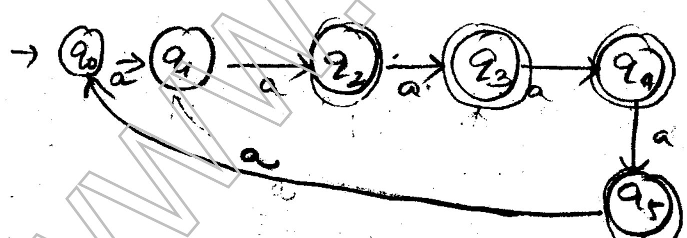
aaaaaaa : $S \to A \to aaaA \to aaaaaaA \to aaaaaaa$

aaaaaaaaa : ×

(c) $\boxed{L = \{ w : |w| \bmod 3 \neq |w| \bmod 2 \}}$

mod 3 → {0, 1, 2} mod 2 → {0, 1}

→ Skip every 6^{th} & 7^{th}



#	mod2	mod3
0	0	0
1	1	1
2	0	2
3	1	0
4	0	1
5	1	2
6	0	0
7	1	1
8	0	2
9	1	0
10	0	1
11	1	2
12	0	0
13	1	1
14	0	2

$S \to aaA$

$A \to \lambda / a / aa / aaa / aaaaS / aaaaaaA$

$S \to aa / aaa / aaaa / aaaaa / aaaaaaS$

(2) (3) (4) (5) (6)+S

test

λ × 9a's ✓
a × 8a's ✓
aa ✓ 13a's ×

$S \to aaA$

$A \to \lambda / a / aa / aaa / aaaaS$

(d) $\boxed{L = \{w : |w| \bmod 3 \geq |w| \bmod 2\}}$

$\|w\|$	mod 2	mod3	$\geq$
0	0	0	✓
1	1	1	✓
2	0	2	✓
3	1	0	✗
4	0	1	✓
5	1	2	✓
6	0	0	✓
7	1	1	✓
8	0	2	✓
9	1	0	✗
10	0	1	✓
11	1	2	✓
12	0	0	✓
13	1	1	✓
14	0	2	✓
15	1	0	✗

$S \to \lambda / a / aa / aaaA$

$A \to a / aa / aaa / aaaa / aaaaa /$
$aaaaaaA$

Test : 3a's : $S \to aaaA$ ✗

5a's : $S \to aaaA \to aaaaa$ ✓

(16) Find a grammar that generates the language $L = \{ww^R : w \in \{a,b\}^+\}$

$\boxed{S \to aSa / bSb / aa / bb / ba}$ $\{a,b\}^+$

abba : $S \to aSa \to abSba \to abba$

bbbb $S \to bSb \to bbSbb \to bbbb$

$S \to aSa / bSb / aa / bb$

(17) Give verbal description of

$S \to aSb / bSa / a$

In no order of a and b, no. of a's are one more in any string.

$S \to aSb \to aaSbb \to aaabb$

$\to bSa \to baa$

$bSa \to bbSaa \to bbaaa$

b's : n

a's : n+1

18

(a) $L = \{w : n_a(w) = n_b(w) + 1\}$

we know for $L_1 = \{w : n_a(w) = n_b(w)\}$ equal a's & b's

$G \Rightarrow S \to SS$

$S \to aSb / bSa / \lambda$

$A \to \underbrace{AA / aAb / bAa / \lambda}_{\text{equal a's \& b's}}$

$S \to \boxed{AaA}$

additional A

$\therefore$ $S \to SSa / aSS / aSb / bSa / \lambda / SaS$

Test:

$S \to SSa \to aSbSa \to a\lambda b\lambda a \to aba \to \boxed{aba \checkmark}$

$S \to \boxed{a \checkmark}$

$S \to SSa \to aSbbSaa \to a\lambda bb\lambda aa \to \boxed{abbaa} \checkmark$

$S \to SSa \to bSaa \to bbSaaa \to bbbSaaaa \to \boxed{bbbaaaa} \checkmark$

$S \to SaS \to bSaaaSb \to \boxed{baaab} \checkmark$

(b) $L = \{w : n_a(w) > n_b(w)\}$ $S \to SS / aSb / bSa / aS / a$

add any no. of a's

$S \to SS / \cancel{aS /} aSb / bSa / \lambda$

(c) $L = \{w : n_a(w) = 2n_b(w)\}$

$S \to SS / aSba / aaSb / bSaa / abSa / aSab / baSa / \lambda$

test: reject aabb : $aaSb \to aab \times$

aaab : $aaSb \to \cancel{aa} \times$

aab : $S \to aab \checkmark$

ababbaaaa : $S \to SS \to abSa \to ababSaa \to ababbaSaaa \to ababbaaaa \checkmark$

aaaaaabbb : $SS \to aaSb \to aaaaSbb \to aaaaaaSbbb \to aaaaaabbb \checkmark$

(d) $L = \{w \in \{a,b\}^* : |n_a(w) - n_b(w)| = 1$

Equal a's & b's

$A \to AA / aAb / bAa / \lambda$

$\Rightarrow n_a(w) = n_b(w) + 1$ ✓

$n_b(w) = n_a(w) + 1$ ✓

$S \to AaA / AbA$

$S \to A / B$

$A \to AAa / aAA / AaA / aAb / bAa / \lambda$

$B \to BBb / bBB / BbB / aBb / bBa / \lambda$

(19) $\Sigma = \{a, b, c\}$

(a) $L = \{w : n_a(w) = n_b(w) + 1\}$

$a = b$: c varying

$S \to SS / aSb / bSa / cS / \lambda$

we know for $\Sigma = \{a, b\}$ $\quad n_a(w) = n_b(w)$

$a = b + 1$: c varying

$S \to SS / aSb / bSa / \lambda$

$S \to AaA$

$A \to AA / aAb / bAa / cA / \lambda$

$\therefore \Sigma = \{a, b, c\}$: $S \to SS / aSb / bSa / C$

$C \to cC / \lambda$

$S \to AaA$

$A \to aAb / bAa / C$

$C \to cC / \lambda$

$\therefore n_a(w) = n_b(w) + 1 \Rightarrow$

$S \to aSS / SSa / SaS / aSb / bSa / C$

$C \to cC / \lambda$

(or) $S \to SSa / aSS / SaS / aSb / bSa / cS / \lambda$

(b) $L = \{w : n_a(w) > n_b(w)\}$

$S \to SS / aSb / bSa / cS / aS / a$

$S \to SS / aS / aSb / bSa / cS / \lambda$

(c) $L = \{w : n_a(w) = 2n_b(w)\}$

$S \to SS / cS / aaSb / aSba / aSab / abSa / baSa / bSaa / \lambda$

(d) $L = \{w : |n_a(w) - n_b(w)| = 1\}$ $\quad$ ~~$S \to S_1 / S_2$~~ add one a / add one b

$S \to AaA / AbA$

$A \to aAb / bAa / AA / cA / \lambda$

$S_2 \to$ ~~$S_2 S_2 / cS_2 / baSb / abSb / bbSa / bSab / bSba / \lambda$~~

20 PT.

$S \to aAb / \lambda$
$A \to aAb / \lambda$ generates $\{a^n b^n : n \geq 0\}$

$S \to \lambda$
$S \to aAb \to ab$
$S \to aAb \to aaAbb \to aabb$

$\therefore L = \{\lambda, ab, aabb \ldots\}$

$L = \{a^n b^n : n \geq 0\}$ is true

21 $S \to aSb / ab / \lambda$ $\equiv$ $S \to aAb / ab$
$A \to aAb / \lambda$

$S \to \lambda$
$S \to ab$
$S \to aSb \to aabb$

$L = \{a^n b^n : n \geq 0\}$

$S \to ab$
$S \to aAb \to ab$
$S \to aaAbb \to aabb$

$L = \{a^n b^n : n > 0\}$

as both grammars represent different languages, they are not equivalent.

22 ST. $S \to SS / SSS / aSb / bSa / \lambda$ is equivalent to $S \to SS / aSb / bSa / \lambda$.

If we rewrite SS as SSS $\{\because S \to SS\}$

both are representing same Grammars.

where $n_a(w) = n_b(w)$

23 ST. $S \to aSb / bSa / SS / a$ $\neq$ $S \to aSb / bSa / a$

$S \to a$
$S \to SS \to aa$

$aa \in L_1$

$S \to aSb \to aab$

$aa \notin L_1$

CHAPTER 1-3

Example 1.15

① id is a sequence of letters, digits, underscores

② id must start with a letter or underscore

③ id allow upper & lower case letters.

<id> → <letter> <rest> / <undscr><rest>

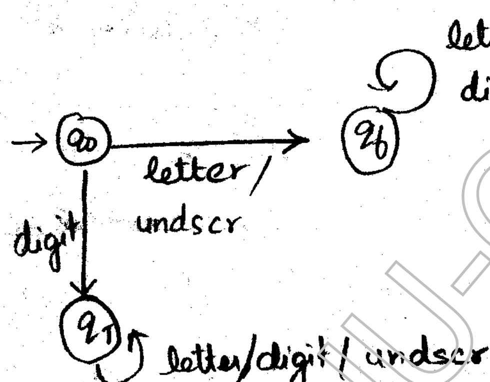
<rest> → <letter><rest> / <digit><rest> / <undscr><rest> / λ

<letter> → a|b|c| - - - z / A|B|C - - Z

<digit> → 0|1|2 - - -|9

<undscr> → _

1.16



Example 1.17

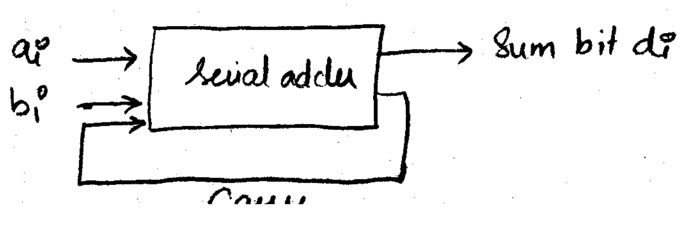
Binary Adder

	b_i = 0	b_i = 1
a_i = 0	0 No Carry	1 No Carry
a_i = 1	1 No Carry	0 Carry

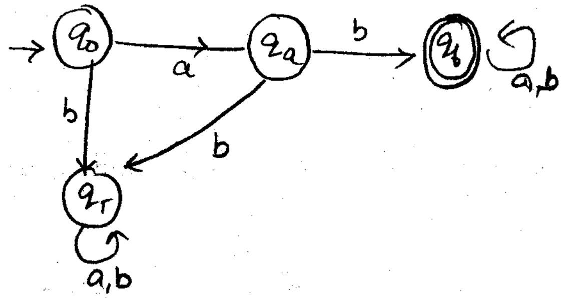
$$v(x) = \sum_{i=0}^{n} a_i 2^i$$

i/p : (a_i, b_i)

o/p = sum bit d_i

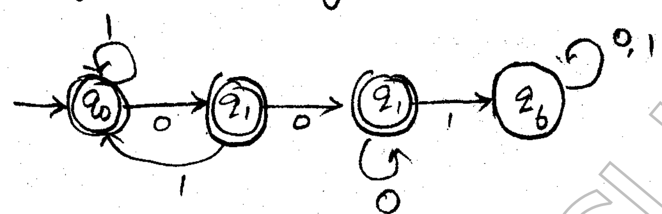


Example 2.3 Find Dfa that recognises all strings on $\Sigma = \{a, b\}$ with prefix ab.



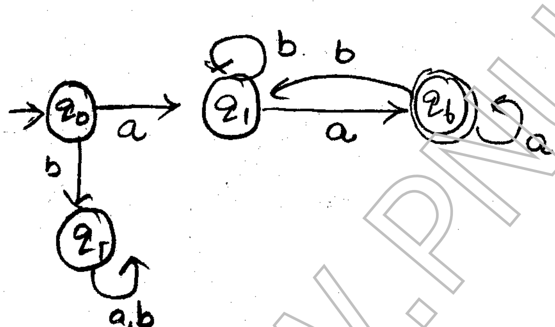
Example 2.4

Find a dfa that accepts all the strings on $\{0,1\}$ except those containing the substring 001.



Example 2.5

Show that $L = \{awa : w \in \{a,b\}^*\}$ is regular.

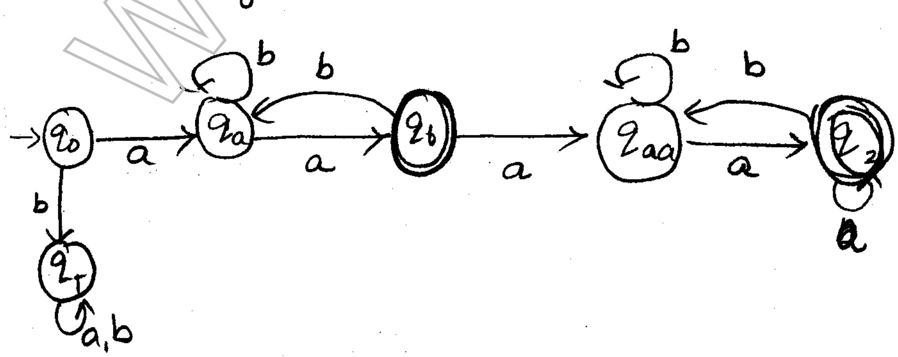


Show that Regular ⇒ Draw Dfa

Example 2.6

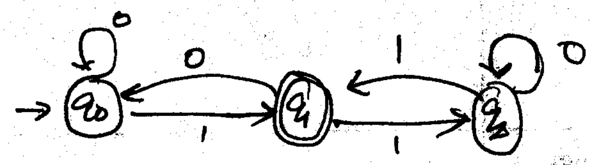
$L = \{aw_1aaw_2a : w_1, w_2 \in \{a,b\}^*\}$ is regular.

L regular ⇒ $L^1, L^2, L^3 \ldots$ are also regular.



EXERCISES

① Which of following are accepted by

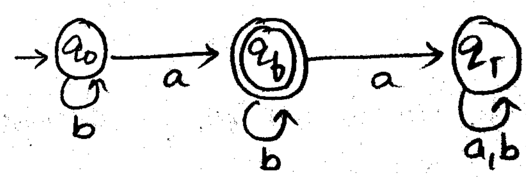


0001 ✓ 0000110. ✗

01001 ✓

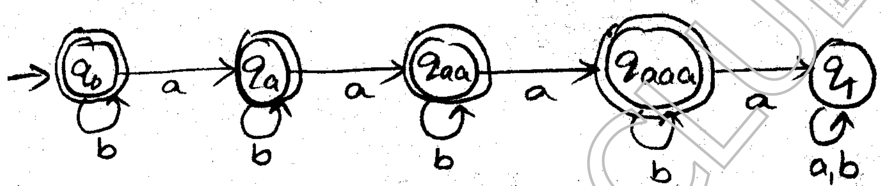
② for $\Sigma = \{a, b\}$ construct dfa's

(a) all strings with exactly one a.

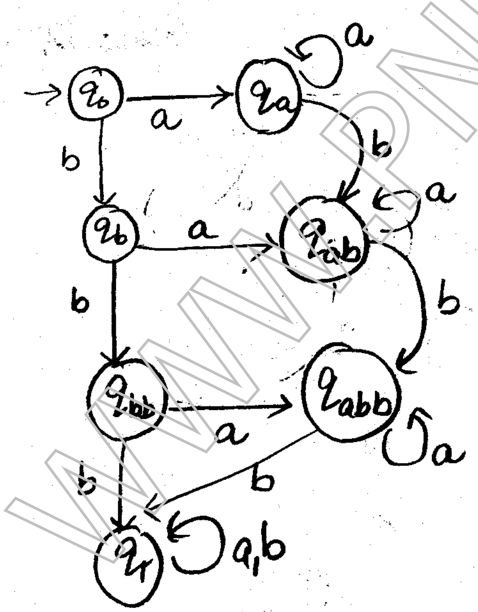


(b) all strings with @least one a

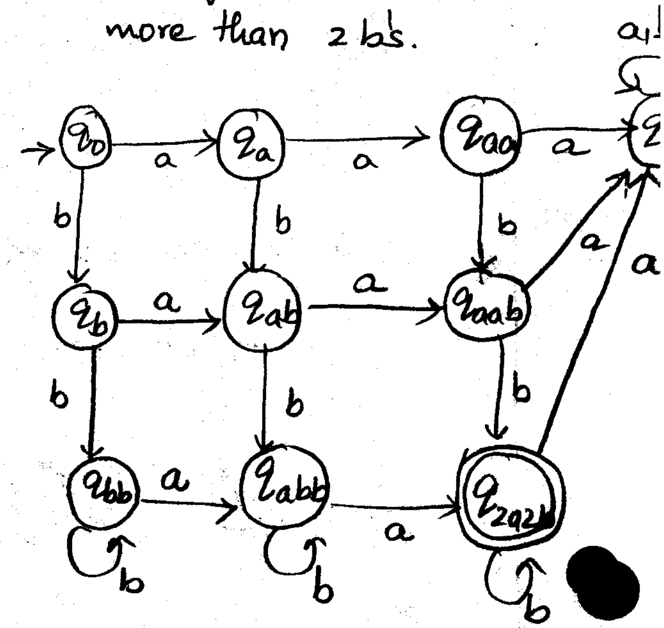
(c) all strings with no more than 3 a's.



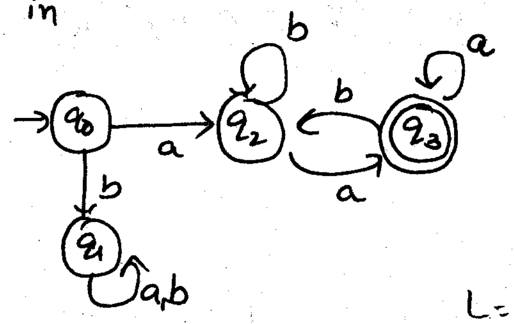
(d) @least one a & exactly 2 b's.



(e) all strings with 2 a's & more than 2 b's.



(3) ST in



If: $q_3 \notin F$

$q_0, q_1, q_2 \in F$, resulting dfa accepts $\bar{L}$.

$L = \{awa : w \in \Sigma^*\}$

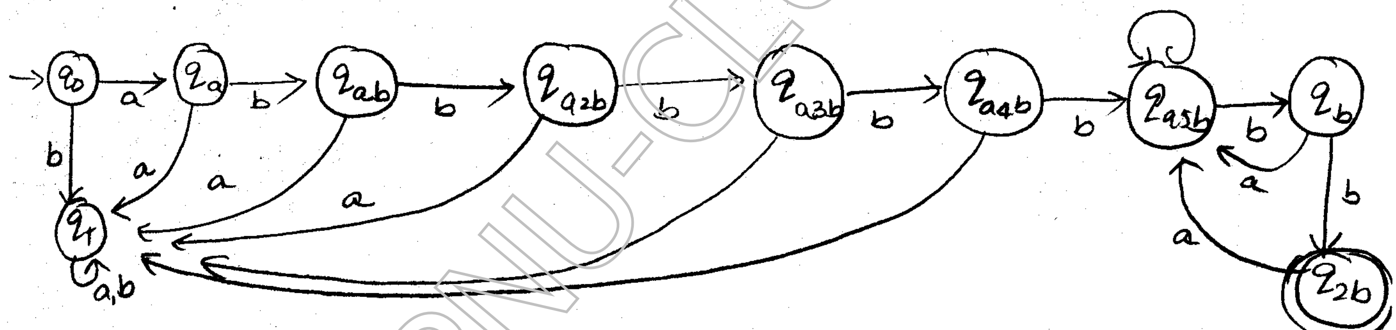
$\bar{L}$: is accepted by the changes to L.

(4) $M = (Q, \Sigma, \delta, q_0, F)$

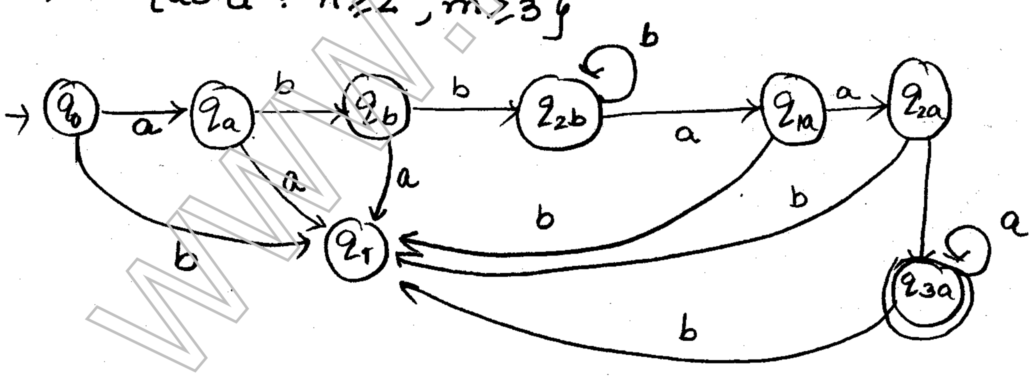
$\hat{M} = (Q, \Sigma, \delta, q_0, Q-F)$

then $\overline{L(M)} = L(\hat{M})$

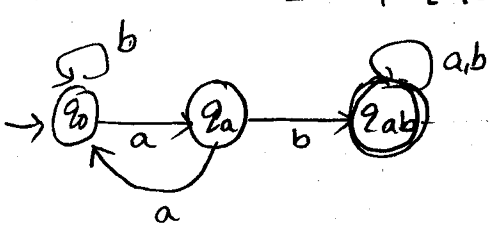
(5) (a) $L = \{ab^5wb^2 : w \in \{a,b\}^*\}$



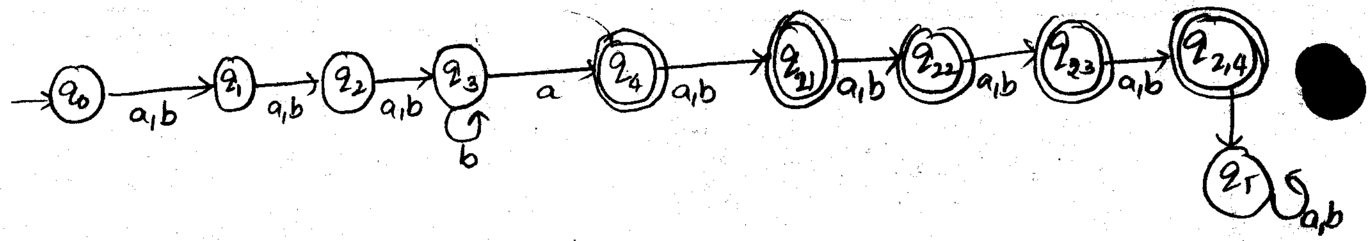
(b) $L = \{ab^na^m : n \geq 2, m \geq 3\}$



(c) $L = \{w_1abw_2 : w_1 \in \{a,b\}^*, w_2 \in \{a,b\}^*\}$

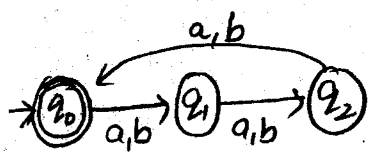


⑥ $\Sigma = \{a,b\}$ give dfa for $L = \{w_1 a w_2 : |w_1| \geq 3, |w_2| \leq 5\}$

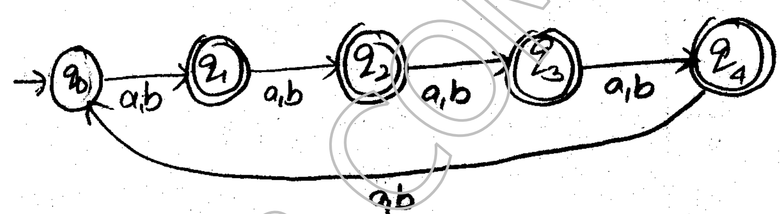


⑦ on $\Sigma = \{a,b\}$

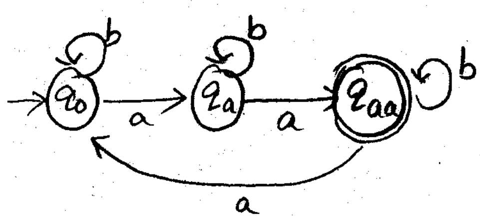
(a) $L = \{w : |w| \bmod 3 = 0\}$



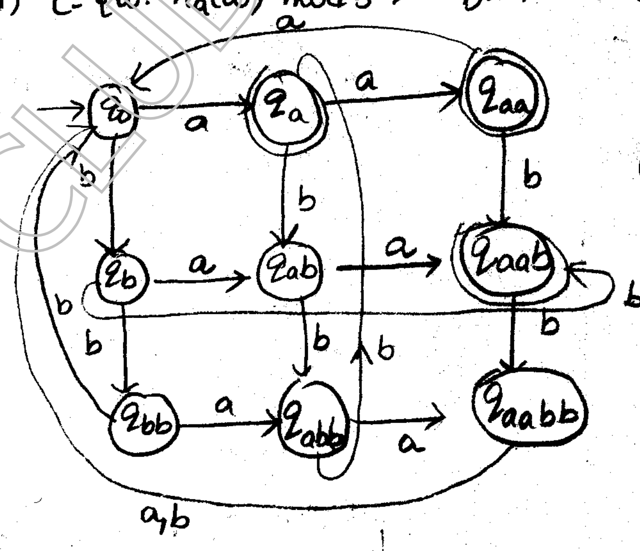
(b) $L = \{w : |w| \bmod 5 \neq 0\}$



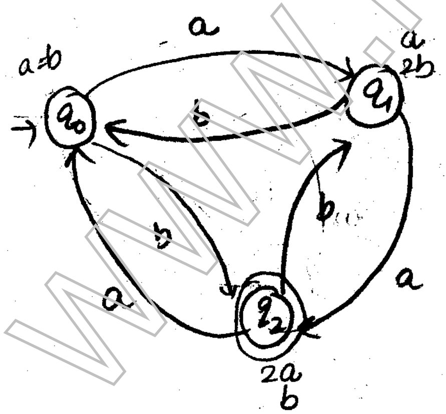
(c) $L = \{w : n_a(w) \bmod 3 > 1\}$



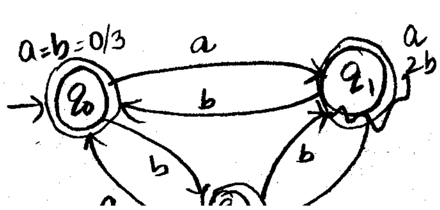
(d) $L = \{w : n_a(w) \bmod 3 > n_b(w) \bmod 3\}$



(e) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 > 0\}$

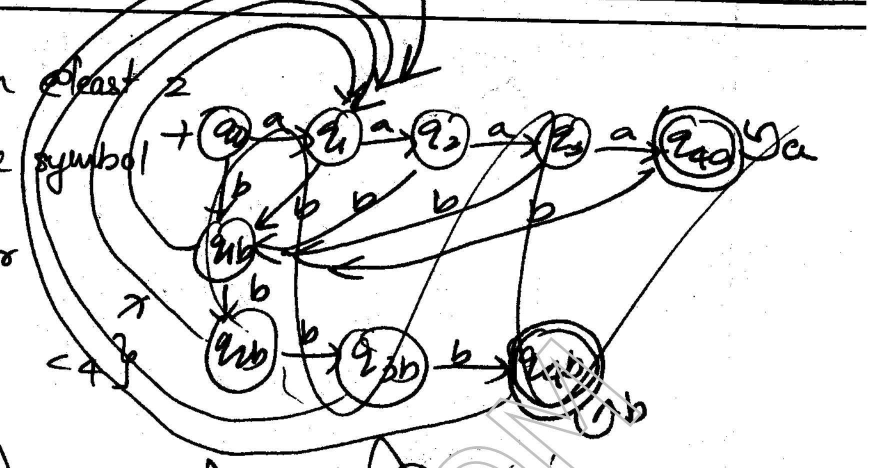


(f) $L = \{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$.

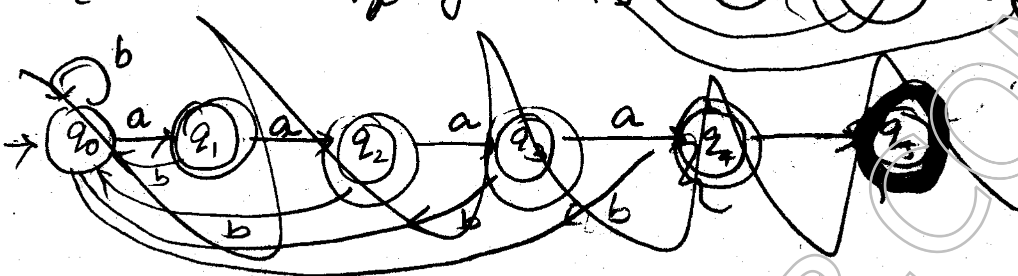


(8) run = substring of length atleast 2
& entirely of same symbol

on $\Sigma = \{a,b\}$ find dfa for

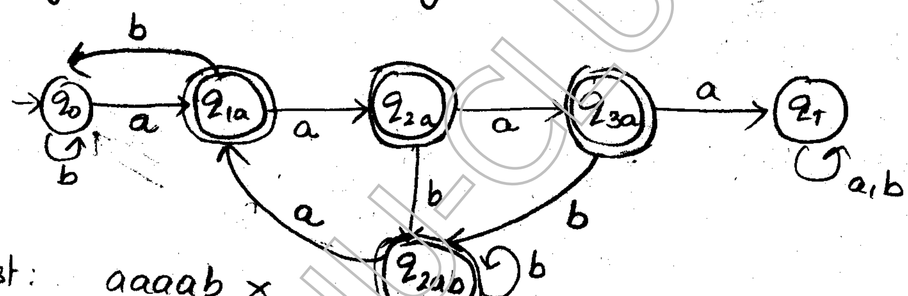


(a) $L = \{w$: no runs of ~~exact~~ length $< 4\}$



(b) $L = \{w$: every run of a's has length 2 or 3. { implies one a accepted }

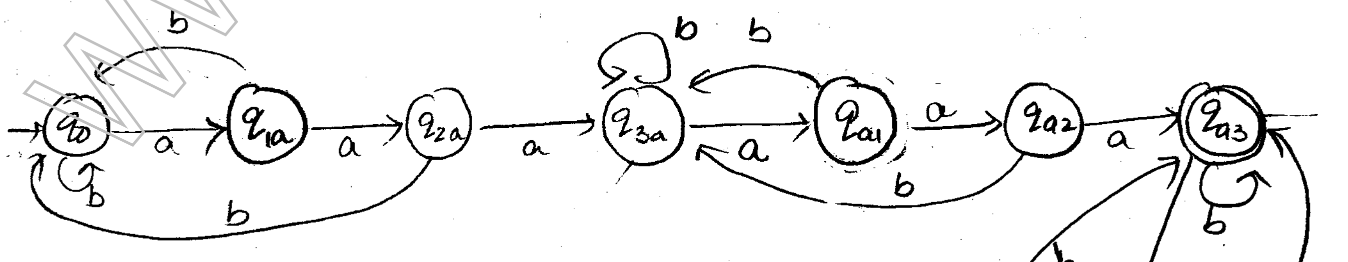
zero, one a should be accepted also.



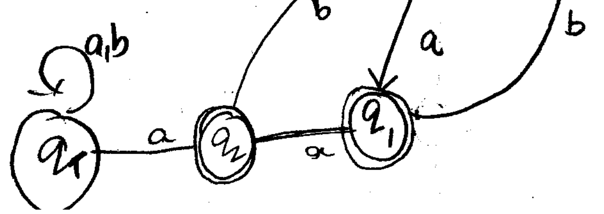
Test: aaaab ×
aabba ✓
baaab ✓
baba ✓
baaaa ×

(d) $L = \{w$: ~~[illegible]~~ exactly 2 runs of a's of length 3.$\}$

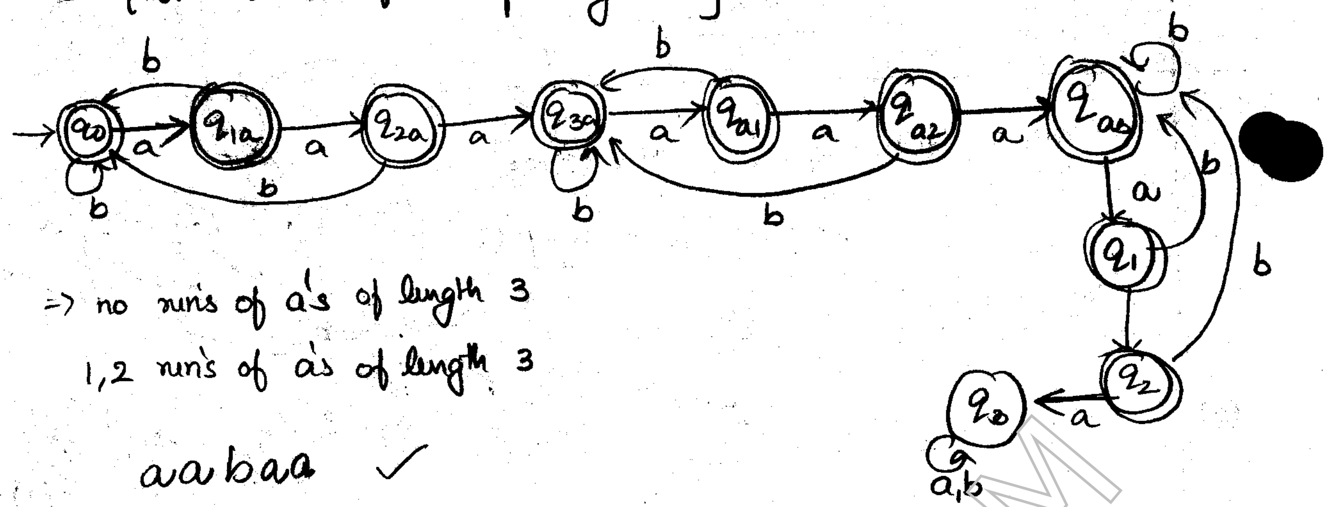
waaaw / waaawaaaw



Test: aaabaaa ✓
aaabaaabaa
aabaaabaaa



(d) $L = \{w :$ @most 2 runs of a's of length 3$\}$



=> no run's of a's of length 3

1,2 run's of a's of length 3

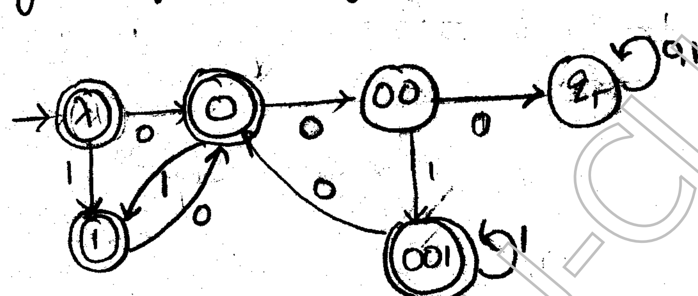
aabaa ✓

aaab

aaabaaa

(9) $\Sigma = \{0,1\}$

(a) Every 00 followed by 1



Test:

00100111 ✓

010001 ✗

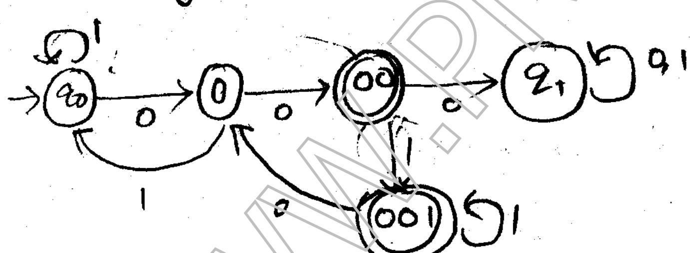
01 ✓

10 ✓

1 ✓

0 ✓

(b) All strings containing 00 but not 000.

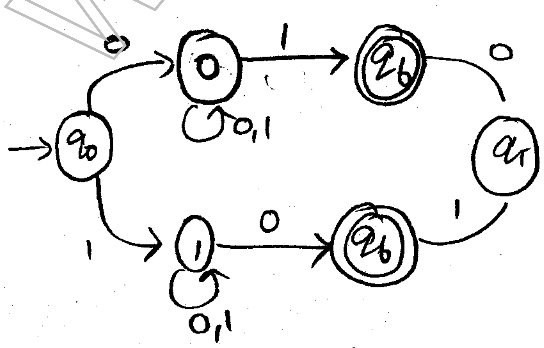


Test: 0010011 ✓

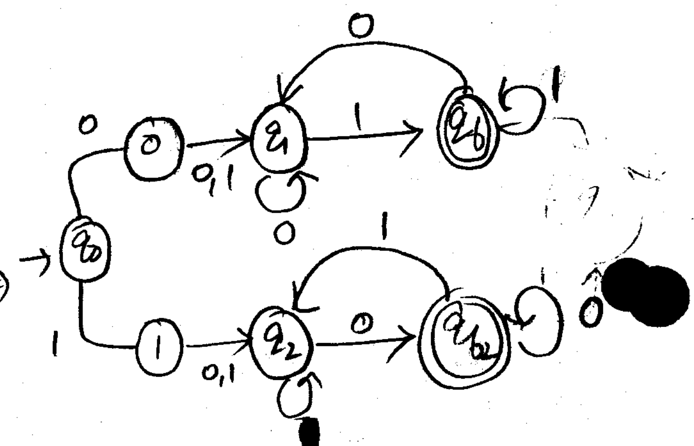
00011 ✓

1001000 ✗

(c) leftmost differs from rightmost.



=>



nfa

1 w 0

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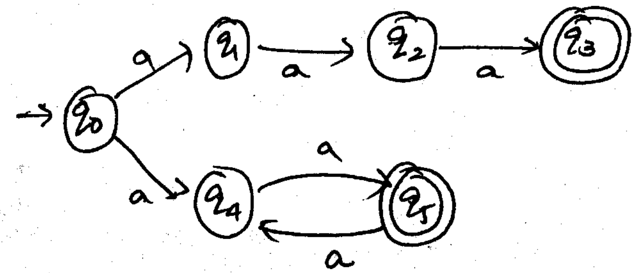
Nfa:

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\delta: Q \times (\Sigma \cup \{\lambda\}) \to 2^Q$$

Example 2.7

fig 2.8



Example 2.9

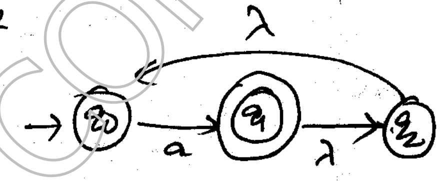
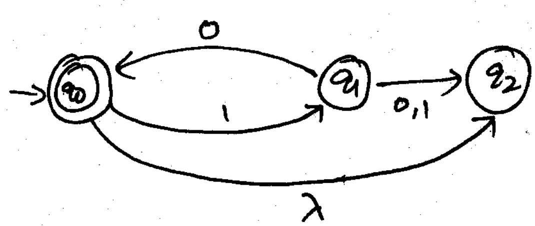


fig 2.10

Example 2.8

fig 2.9

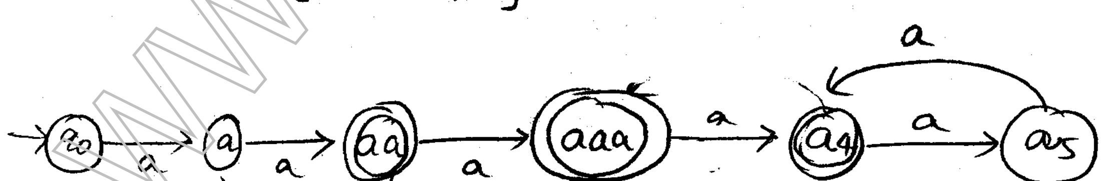


$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \emptyset\}$$

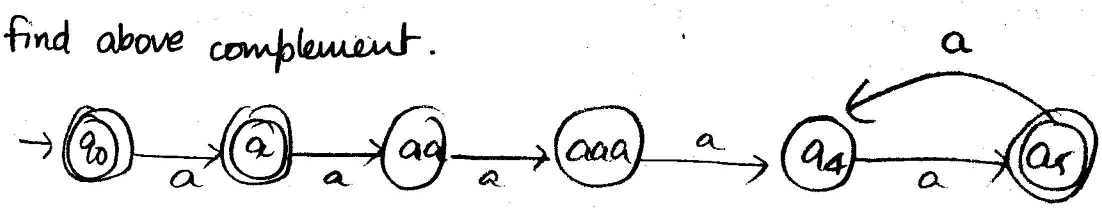
EXERCISES

(2) find dfa defined by : fig 2.8

$L: \{aaa\} \cup \{a^{2n} : n \geq 1\}$



(3) find above complement.



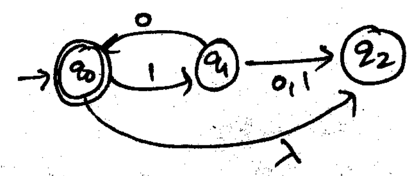
④ Fig 2.9 $\delta^*(q_0, 1011)$: $\delta^*(q_1, 011) \rightarrow \delta^*(q_0, 11) \rightarrow q_2$

$\delta^*(q_1, 01)$: q_2

⑤ Fig 2.10 : $\delta^*(q_0, a)$ $\{q_0, q_1, q_2\}$

$\delta^*(q_1, \lambda)$ $\{q_0, q_2\}$

⑥ for:



$\delta^*(q_0, 1010) \rightarrow \delta^*(q_1, 010) \rightarrow \delta^*(q_0, 10) \rightarrow \delta^*(q_1, 0) \rightarrow q_2$ $\{\cancel{q_0}\, q_2\}$

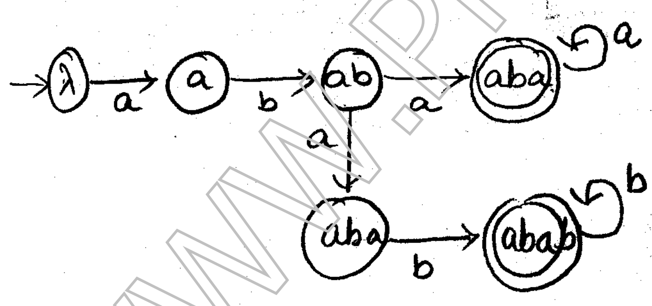
$\delta^*(q_1, 00)$ $\rightarrow \delta^*(q_2, 1010)$ × $\rightarrow \delta^*(q_2, 10)$ × $\delta^*(q_2, 10)$ ×

↳ $\delta^*(q_1, 00) \rightarrow \delta^*(q_0, 0) \rightarrow q_2$

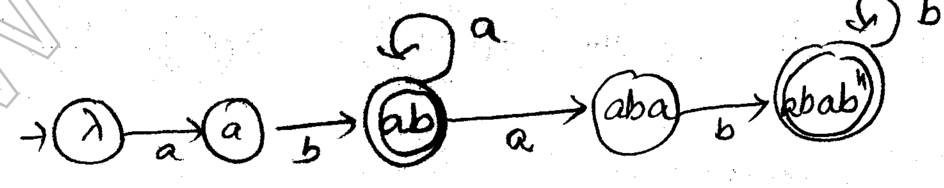
$\delta^*(q_2, 0)$ ×

⑦ no more than 5 states, design nfa for

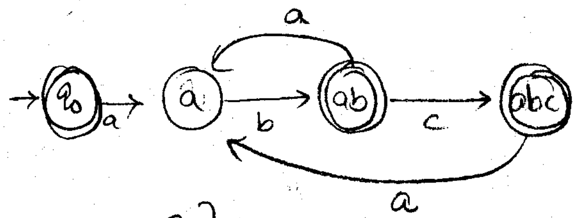
$\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$

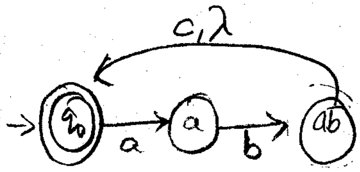


$\Rightarrow$



(8) Construct nfa with 3 states for $\{ab, abc\}^*$





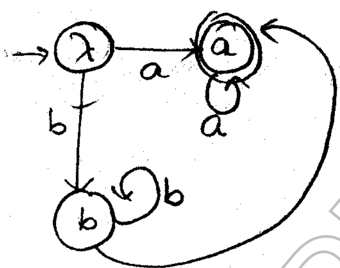
(9) Can it be done in fewer states? than 3?

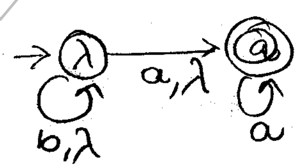
No as $|abab^n|$ least $= 3$ for $n=0$

(10) (a) find nfa with 3 states that accepts

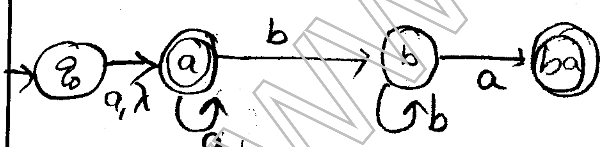
$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}$

(b) can fewer than 3 states be possible?

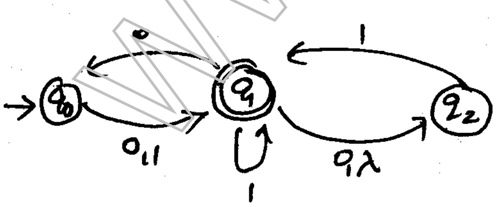




(11) Nfa - 4 states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$



(12)



00 : $\delta^*(q_0, 00) \rightarrow \{q_0\ q_2\} \cap F = \emptyset$ reject

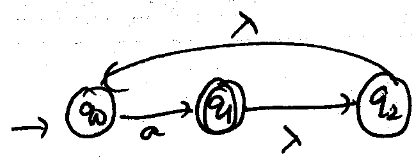
01001 : $\{q_1\} \cap F \neq \emptyset$ accept

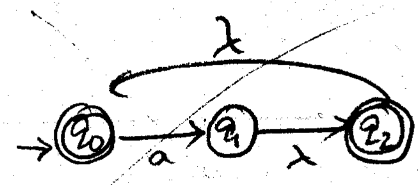
10010 : $\{q_0, q_2\} \cap F = \emptyset$ reject

000 : $\{q_1, q_2\} \cap F \neq 0$ accept

0000 : $\{q_0\ q_2\}$ reject

(13) What is complement of

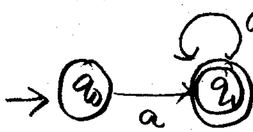




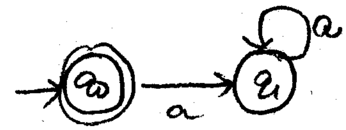
$\{\lambda, a, aa, \ldots\}$

can't Take complement of nfa

dfa



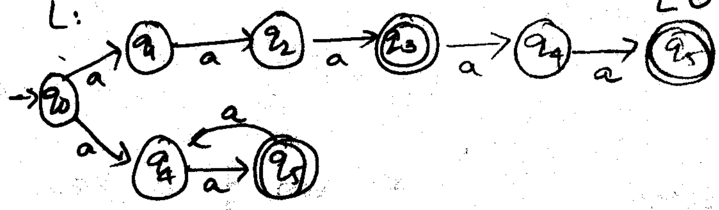
complement $\longrightarrow$



all strings $n_a(w) > 1$

all strings $n_a(w) < 1$

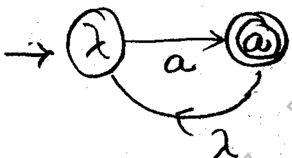
(14) L:



$L \cup \{a^5\}$?

(15) 13 ? $\{a\}^* - \{a\}^+$

(16) Find nfa for $\{a\}^*$ such that removing one edge will accept $\{a\}$



(17) Can above be done with dfa ?

No : need two paths to accept one a / more a's

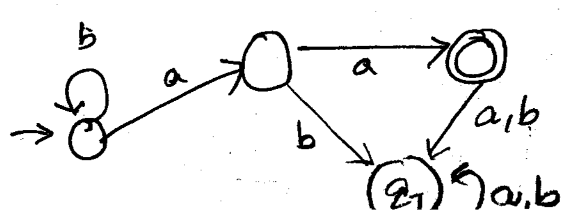
(18) let $p_0 \in Q_0$.

$\delta(p_0, \lambda) \rightarrow Q_0$

if Q_0 not initial, equivalent to M.

(19) Yes.

(20)

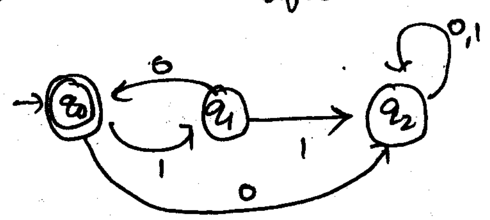


CH 2-3

Equivalence of Nfa & Dfa:

fig 2.11

dfa

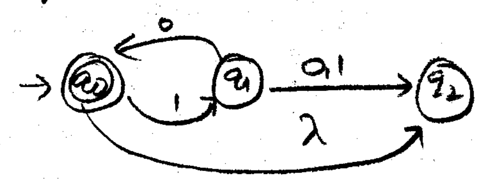


$\{\lambda, 10, 1010 \ldots\}$

$\Rightarrow \{(10)^n : n \geq 0\}$

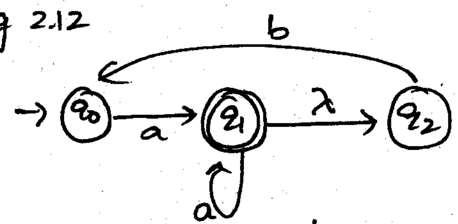
$\simeq$

nfa



$\{(10)^n : n \geq 0\}$

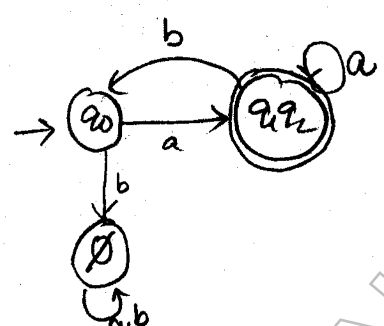
fig 2.12



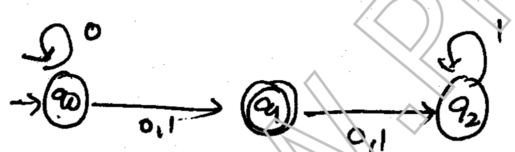
nfa

	a	b
$\{q_0\}$	$\{q_1, q_2\}$	$\emptyset$
F $\{q_1 q_2\}$	$\{q_1, q_2\}$	$\{q_0\}$
$\emptyset$	$\emptyset$	$\emptyset$

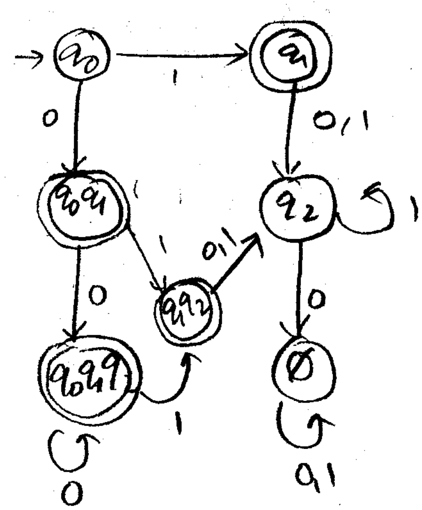
dfa



Example 2.13



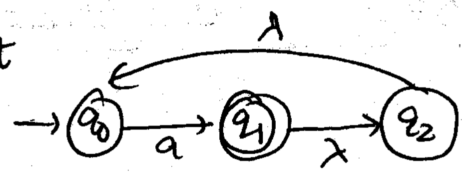
	0	1
$\{q_0\}$	$\{q_0\ q_1\}$	$\{q_1\}$
$\{q_0 q_1\}$	$\{q_0\ q_1 q_2\}$	$\{q_1,\ q_2\}$
$\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$	$\emptyset$	$\{q_2\}$
$\{q_1 q_2\}$	$\{q_2\}$	$\{q_2\}$
$\{q_0 q_1 q_2\}$	[illegible]	[illegible]



If too many states getting combined, dont end, go on & enumerate all

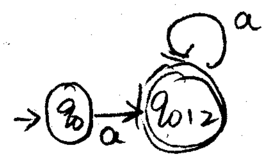
2-3 EXERCISES

(1) Convert

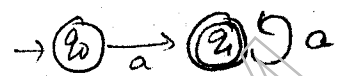


to dfa. Is there a simpler way?

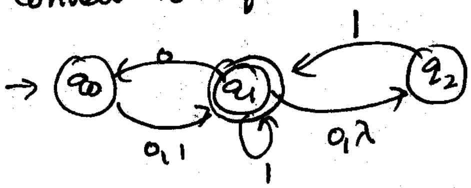
	a	1
$\{q_0\}$	$\{q_0 q_1 q_2\}$	
$\{q_0 q_1 q_2\}$	$\{q_0 q_1 q_2\}$	



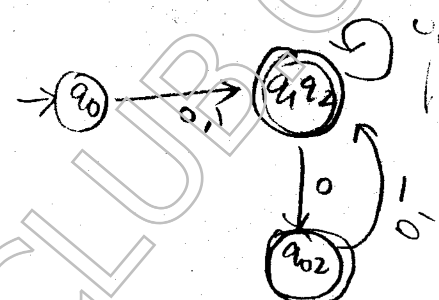
Yes: directly as $L = \{a^n : n > 0\}$



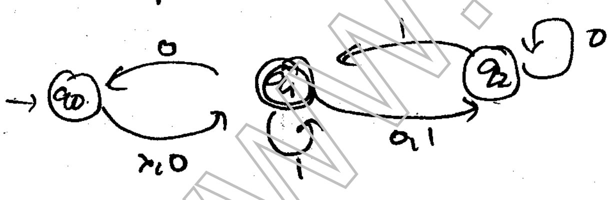
(2) Convert to dfa



	0	1
$\{q_0\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$+\{q_1 q_2\}$	$\{q_0 q_2\}$	$\{q_1, q_2\}$
$\{q_0 q_2\}$	$\{q_1 q_2\}$	$\{q_1 q_2\}$

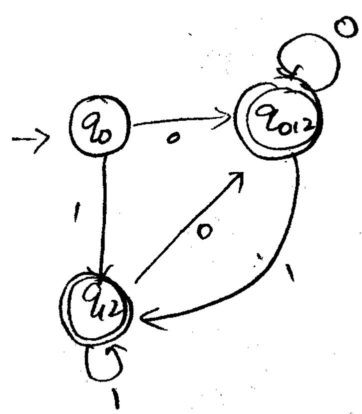


(3) Convert nfa → dfa



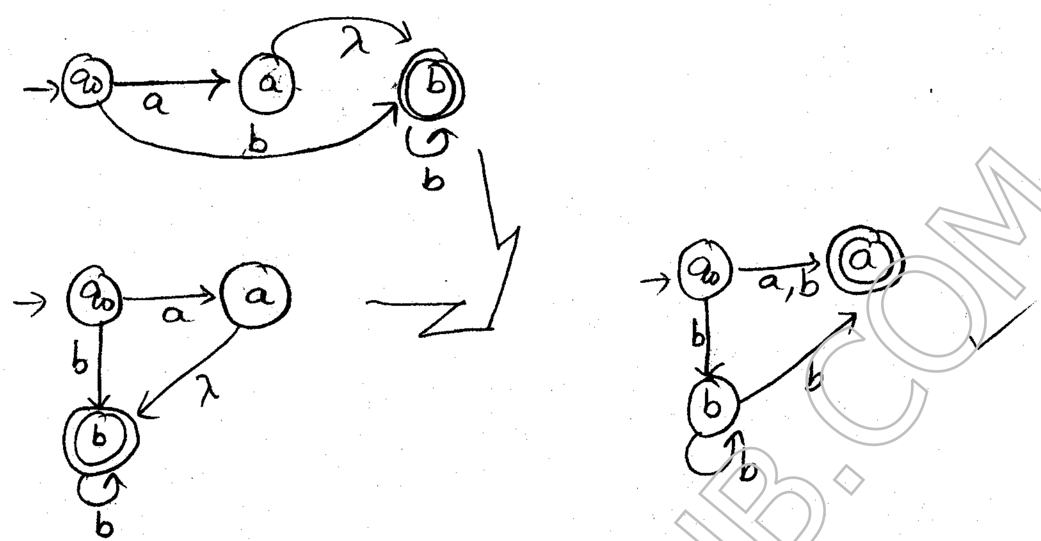
$q_0 = q_1$

	0	1
$\{q_0\}$	$\{q_0 q_1 q_2\}$	$\{q_1 q_2\}$
$\{q_1 q_2\}$	$\{q_0 q_1 q_2\}$	$\{q_1 q_2\}$
$\{q_0 q_1 q_2\}$	$\{q_0 q_1 q_2\}$	$\{q_1 q_2\}$



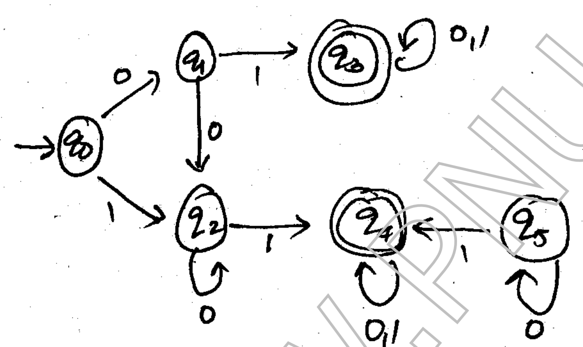
⑧ find nfa without λ-transitions, single final state for

$\{a\} \cup \{b^n : n \geq 1\}$



CH # 2.4 (Reduction of states in dfa)

Example 2.17



	q_0	q_1	q_2	q_3	q_4
q_0		D	D	D	D
q_1			ID	D	D
q_2				D	D
q_3					ID
q_4					

I states: q_5

D: $\{q_0, q_1\}$
$\{q_0, q_2\}$

ID: $\{q_3\ q_4\}$
$\{q_1\ q_2\}$

$\{q_0\}, \{q_1 q_2\}, \{q_3\ q_4\}$

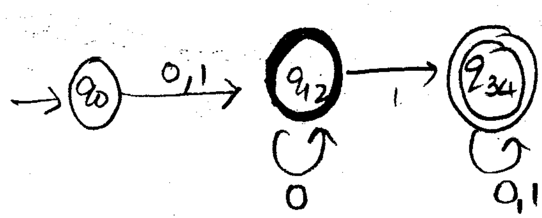
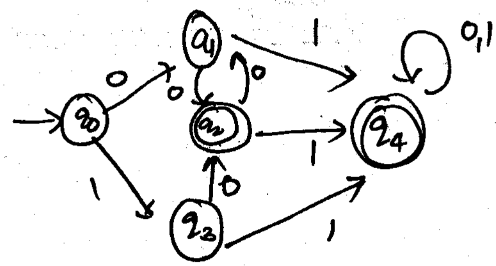


fig: 2.18

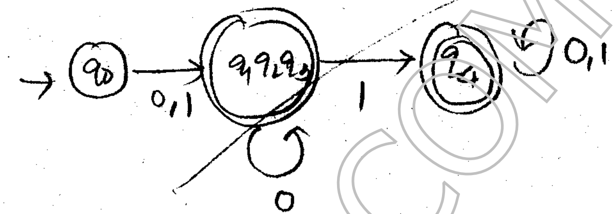


D: $\{q_1\ q_4\}$ $\{q_2, q_4\}$ $\{q_3, q_4\}$ $\{q_0, q_4\}$

ID: $\{q_1, q_3\}$ $\xrightarrow{0}$ $\{q_2\}$ $\{q_4\}$

	q_0	q_1	q_2	q_3	q_4
q_0		D	D	D	D
q_1			ID	ID	D
q_2				ID	D
q_3					D
q_4					

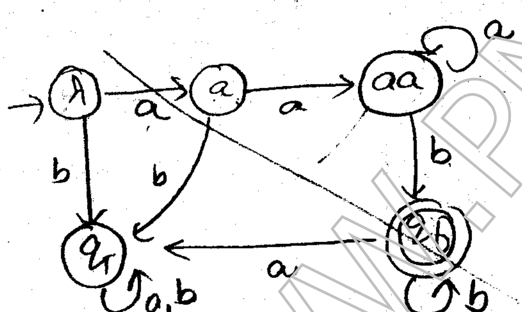
$\{q_0\}, \{q_1 q_2 q_3\}\ \{q_4\}$



EXERCISES

(2) (a) find minimal dfa for

$L = \{a^n b^m : n \geq 2, m \geq 1\}$

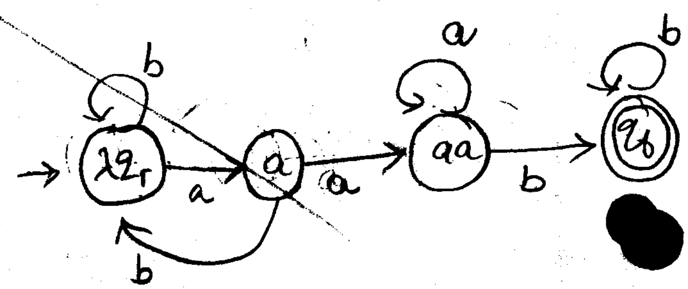


	λ	a	aa	q_f	q_T
λ		D	D	D	ID
a			D	D	D
aa				ID	D
q_f					D
q_T					

$\{\lambda, q_T\}, \{a\}, \{a\}, \{aa\}\ \{q_f\}$

ID: (aa, q_f)
(λ, q_T)

D: (a, q_f) (λ, q_f) $(q_T\ aa)$
(q_T, q_f) (λ, aa) (q_f, aa)



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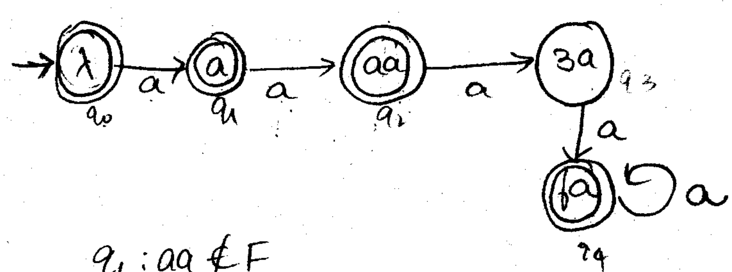
② (a) $L = \{a^n b^m : n \geq 2, m \geq 1\}$

minimal

(b) $L = \{a^n b : n \geq 0\} \cup \{b^n a : n \geq 1\}$

(c) $L = \{a^n : n \geq 0, n \neq 3\}$



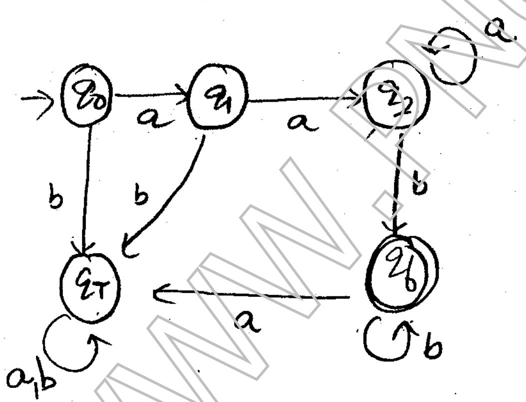
	λ	a	aa	3a	fa
λ		D	D	D	D
a			D	D	D
aa				D	D
3a					D
fa					

$q_1 : aa \notin F$
$(q_4, aa) \in F$ $\therefore (q_1 q_4)$ D

$(\lambda, aaa) \notin F$ $\therefore (\lambda, 3a)$, D
$(3a, aaa) \notin F$

already in minimal //

(a)



	q_0	q_1	q_2	q_f	q_T
q_0		D	D	D	D
q_1			D	D	D
q_2				D	D
q_f					D
q_T					

$q_f \in F$ $\therefore$ D-states
rest $\notin F$

$\therefore$ minimal

CHAPTER : 3

RL & RG

$\rightarrow$ $L(r_1+r_2) = L(r_1) \cup L(r_2)$

$L(r_1 . r_2) = L(r_1) . L(r_2)$

$L(r_1^*) = (L(r_1))^*$

Example 3.2 $L(a^* . (a+b))$?

$L(a^*.(a+b)) = L(a^*) . L(a+b)$

$= L(a^*) . \{L(a) \cup L(b)\}$

$= L(a^*) . \{a, b\}$

$= \{\lambda, a, aa \ldots\} . \{a, b\}$

$= \{a\}^+ \cup \{a^n b^{n+1} : n \geq 0\}$

Example 3.3 $\Sigma = \{a, b\}$

$r = (a+b)^* . (a+bb)$

$\{a,b\}^* . \{a, bb\}$ $= \{a, bb, aa, abb, ba, bbb \ldots\}$

Example 3.4

$r = (aa)^* (bb)^* b$

$L = \{(aa)^n (bb)^n b : n \geq 0\}$

$L = \{a^{2n} b^{2m+1} : n, m \geq 0\}$

Example 3.5 $\Sigma = \{0,1\}$: w has atleast one pair of consecutive zeroes.

$(0+1)^* 00 (0+1)^*$

3.6. no consecutive zeroes : $(1+01)^* (0+\lambda)$

RG, RL's

EXERCISES

① $L[(a+b)^* b (a+ab)^*]$ find strings $|w| < 4$.

$\{\lambda, a, b, ab, ba, aa, bb, aba, baa, aaa, bba, bab, abb, aab, \ldots\}.b.$

$\{\lambda, a, ab, aab, aba, aaa\} \ldots$

$|w|<4$: $\{b, ab, bb, ba, bab \ldots\}$

② $((0+1)(0+1)^*)^* 00 (0+1)^*$ denote @least one pair of consecutive 0's.

yes.

③ $r = (1+01)^* (0+1^*)$ also denotes no consecutive zeroes.

$\downarrow$

$(1+01)^* (0+\lambda+\{1\}^+)$

$((1+01)^* (0+\lambda)) + (1+01)^* \{1\}^+$

$\downarrow$

$(1+01)^*$

$= (1+01)^* (0+\lambda) \Rightarrow$ no consecutive zeroe's

④ RE? $\{a^n b^m : n \geq 3, m \text{ is even}\}$

$aaa(a^*)(bb)^*$

⑤ RE = ? $\{a^n b^m : (n+m) \text{ is even}\}$

$[(aa)^*(bb)^* + (aa)^* a (bb)^* b]$

RE = ?

(6) (a)

$$L_1 = \{a^n b^m : n \geq 4,\ m \leq 3\}$$

$$aaaa \cdot a^* (\lambda + b + bb + bbb)$$

(b) $L_2 = \{a^n b^m : n < 4;\ m \leq 3\}$

$$(\lambda + a + aa + aaa)(\lambda + b + bb + bbb)$$

(c) $\overline{L_1} : \{a^n b^m : n < 4,\ m > 3\}$ (d)

$\times$ $(\lambda + a + aa + aaa)\, bbbb\, b^* +$

RE($\overline{L}$) = all possible rule breakers in RE(L) →

either $n < 4$ or $m \geq 4$ (or) $a^n b^k a^l$

$$(\lambda + a + aa + aaa)\, b^* + a^* bbbbb^* +$$

$$(a+b)^* ba (a+b)^*$$

(d) $\overline{L_2} : \{a^n b^m : \cancel{n < 4}, m \leq 3\}$

$n \geq 4$ / $m > 3$

$$\overline{L_2} : aaaaa^* b^* + a^* bbbbb^* + (a+b)^* ba (a+b)^*$$

(8) $L\,[(aa)^* b (aa)^* + a(aa)^* ba(aa)^*]$

wbw : $w : a^{2n} : n \geq 0$

$w : a^{2n+1}\ \ n \geq 0$

b having even a's on both ends or

b having odd a's on both ends.

3-1

Exercises :

(16)

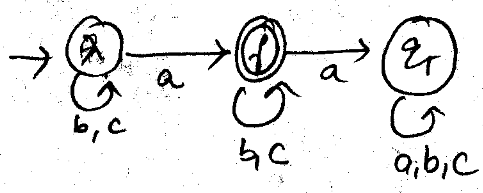
(a) $\Sigma = \{a, b, c\}$

Exactly one a.

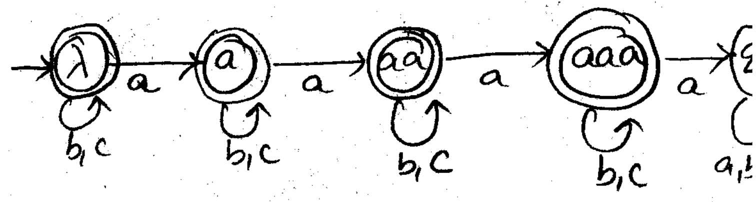
$(b+c)^* a (b+c)^*$

a ✓ bcbca ✓

ab ✓ bcaa ✗



(b) no more than 3 a's.



$[(b+c)^* a (b+c)^* a (b+c)^* a (b+c)^*$

$+ (b+c)^* + (b+c)^* a (b+c)^* +$

$(b+c)^* a (b+c)^* a (b+c)^*]$

Test. aa ✓ aaaa ✗

bca ✓

(c) @least one occurance of each symbol in Σ

$(a+b+c)^* a (a+b+c)^* b (a+b+c)^* c (a+b+c)^*$

(d) no run of a's $|w|_a > 2$

0, 1, 2

$(\lambda + a + aa + b + c)^*$

$\cancel{(b+c)^* (\lambda+a)(b+c)^* (\lambda+a)(b+c)^*}$

(e) run's of a's are multiples of 3.

$\cancel{(b+c)^* aaaa^* (b+c)^*}$ $aaaa^* (b+c)^*$

$(\lambda + aaaa^* + b + c)^*$

17.

(a) Ending in 01

$(0+1)^* 01$

(17)

(b) Not Ending in 01:

$1^* (0 + 01 + 11)^* 11^* (0 + \lambda)$ ✓

(c) Even no. of zeroes

$[1^* 01^* 01^* + 1]^*$

CHAPTER 3-2

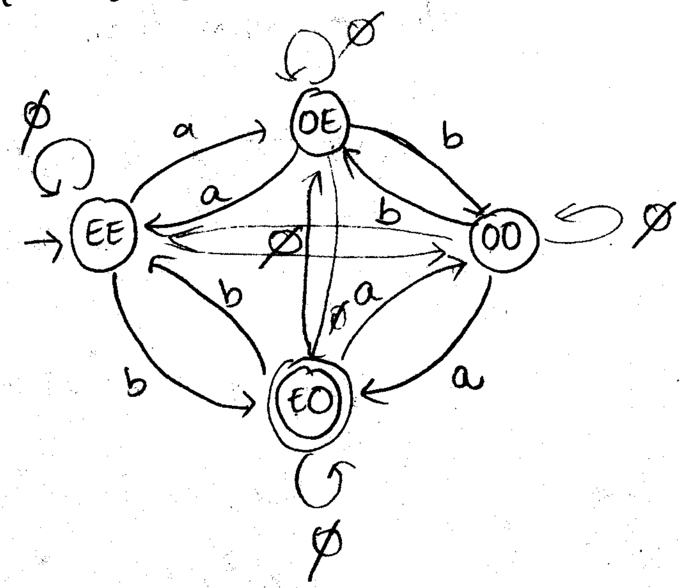
Example 3.11

RE = ?

$L = \{ w \in \{a,b\}^* : n_a(w) \text{ is even}, n_b(w) \text{ is odd} \}$

FIND RE for a nfa?

$\boxed{\emptyset \text{ to all unknown moves!}}$



$r + \emptyset = r$

$r\emptyset = \emptyset$

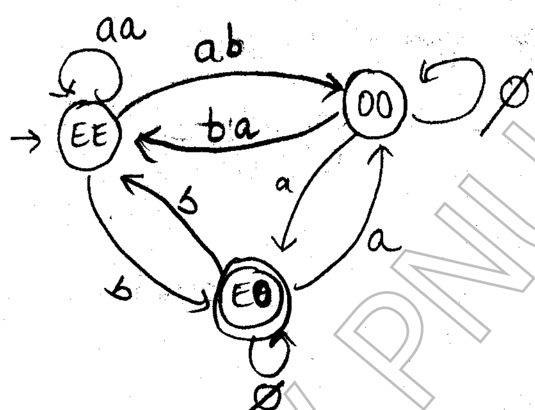
$\emptyset^* = \lambda$

step ①

$r_{EE} = \emptyset + a\emptyset^* a$

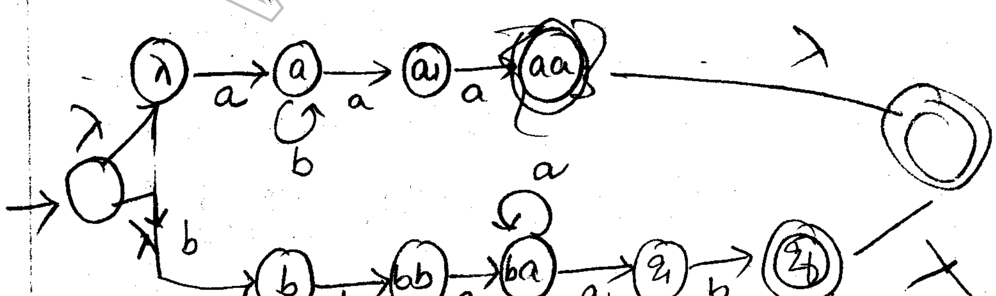
$= aa$

step 2 — now 3-state Rule

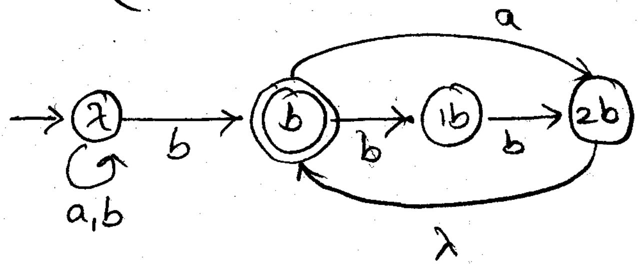


EXERCISE

$L(ab^*aa + bba^*ab)$:

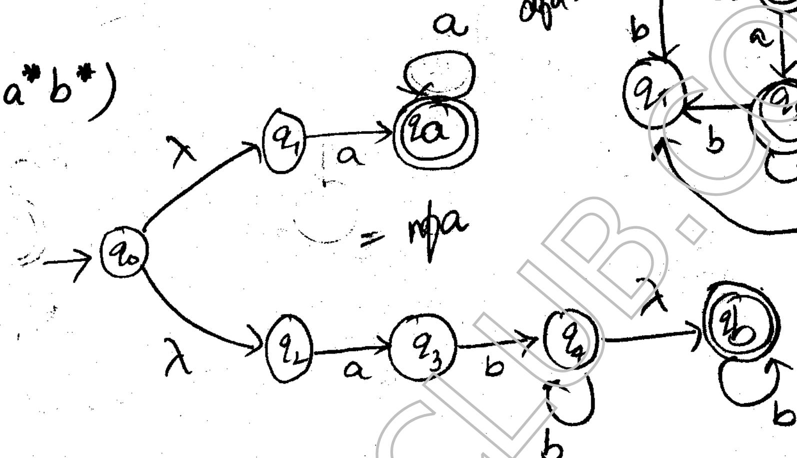


(3) nfa? $((a+b)^* b (a+bb)^*)$

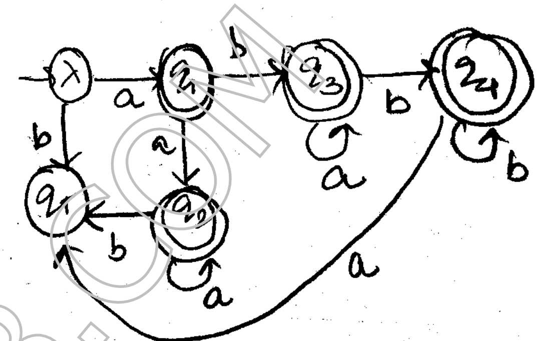


(4) (a) dfa?

$L(aa^* + aba^*b^*)$

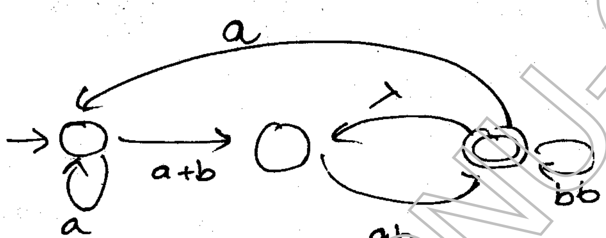


dfa:

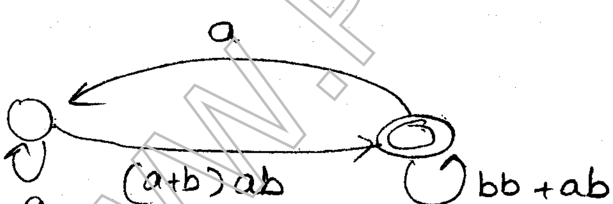


Removing an edge in a transition graph ($q_0 \notin F$)

(8)

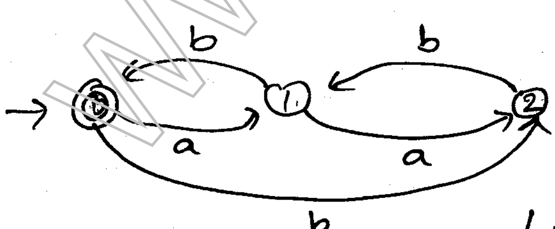


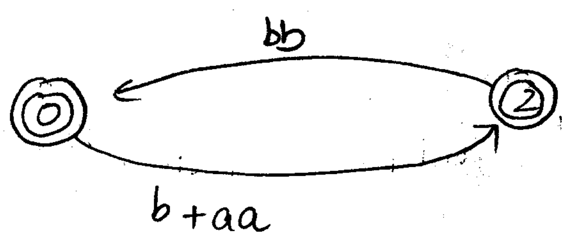
$r = a^* (a+b)ab(ab+bb+aa^*(a+b)ab)^*$



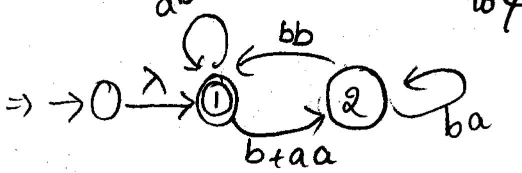
aabbbbb

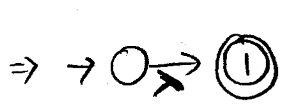
(10) (b)





$q_0 \notin F.$

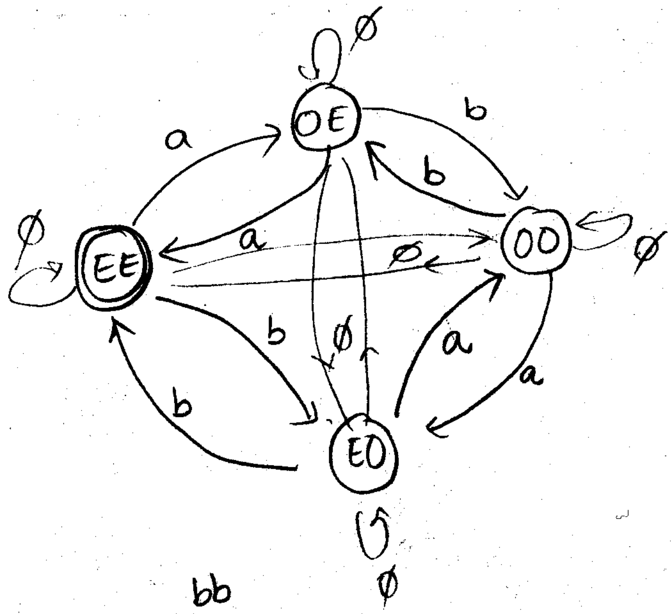


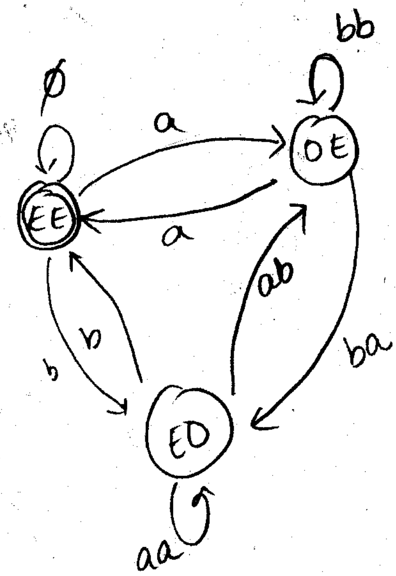


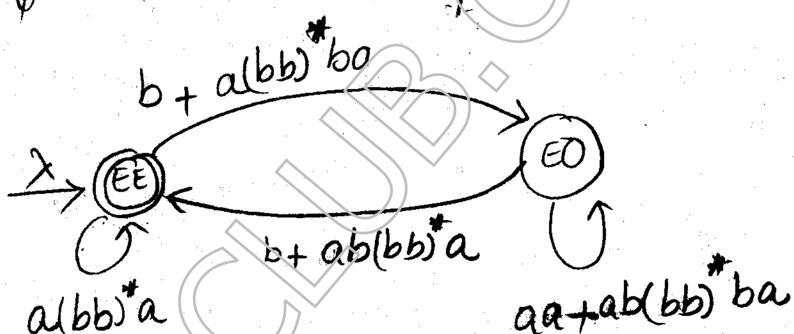
$ab + (b+aa)(ba)^* bb$

(13) RE = ? on $\Sigma = \{a, b\}$

(a) $L = \{w : n_a(w), n_b(w) \text{ are even}\}$







RE :

$\lambda + [a(bb)^*a][b + a(bb)^*ba][aa + ab(bb)^*ba]^*[b + ab(bb)^*a$

λ ✓

aa

CH # 3.3

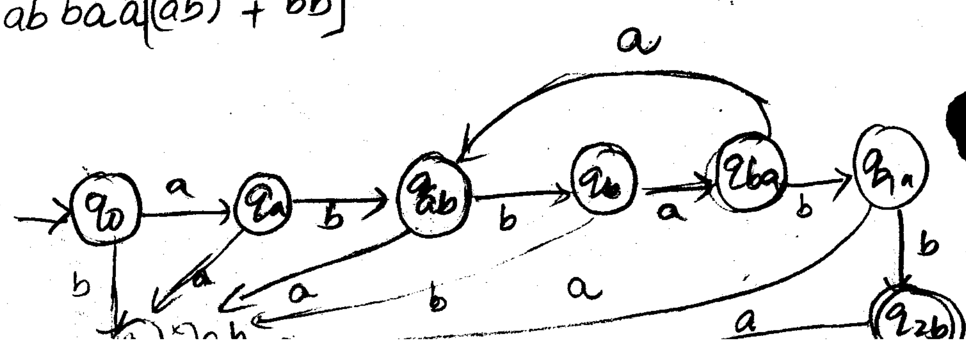
(1) dfa = ?

$S \to abA$

$A \to baB$

$B \to aA / bb$

$ab\,baa[(ab)^* + bb]$



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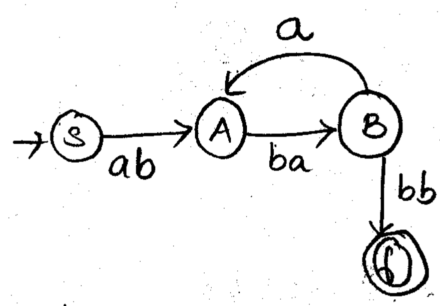
(3) LG = ? for 1

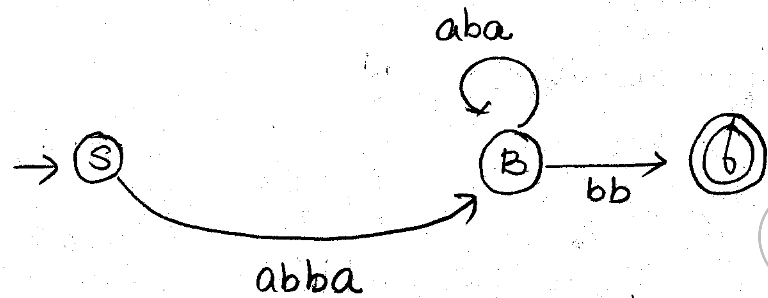
S → abA

A → baB

B → aA / bb

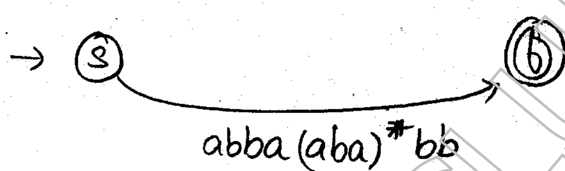
RE =





Test: abbabb ✓

abbaabaababb ✓



$$\boxed{\begin{array}{l} S \to ABbb \\ A \to abba \\ B \to Baba \;/\; \lambda \end{array}}$$

S → ABbb → abbaBbb →

abba Baba bb →

abba(aba)* bb ✓

(4) RLG, LLG = ?

$\{a^n b^m : n \geq 2, m \geq 3\}$

LG

S → aaA bbbB

A → aA / λ

B → bB / λ

RLG:

S → aaAB

A → aA / λ

B → bbbC

C → bC / λ

LLG:

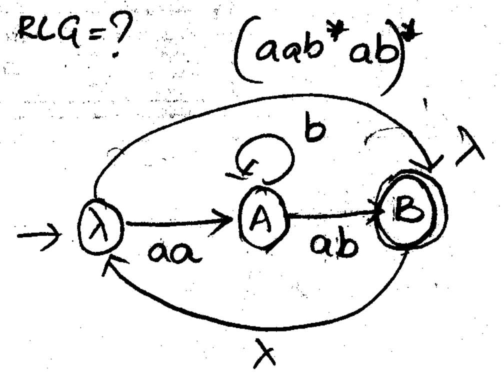
S → ABbbb

A → Caa

C → Ca / λ

B → Bb / λ

(6) RLG = ?



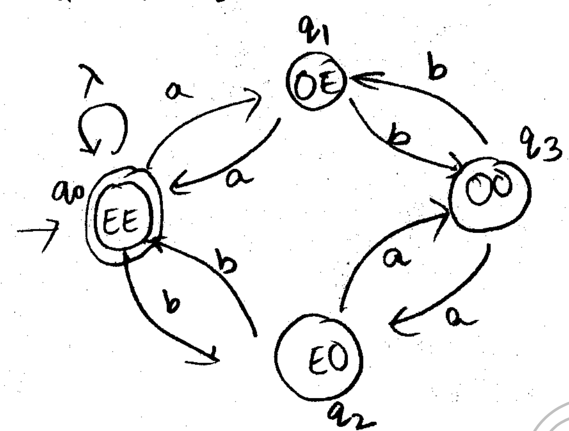
$S \to aaA / \lambda$

$A \to bA / abS$

(13)

(a) ~~REG~~ RG = ? $\Sigma = \{a, b\}$

(a) $n_a(w)$, $n_b(w)$ are even.



$q_0 \to aq_1 / \lambda / bq_2$

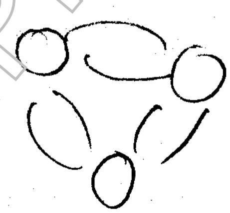
$q_1 \to bq_3 / aq_0$

$q_2 \to aq_3 / bq_0$

$q_3 \to aq_2 / bq_1$

$(n_a - n_b) \bmod 3 = 1$

draw



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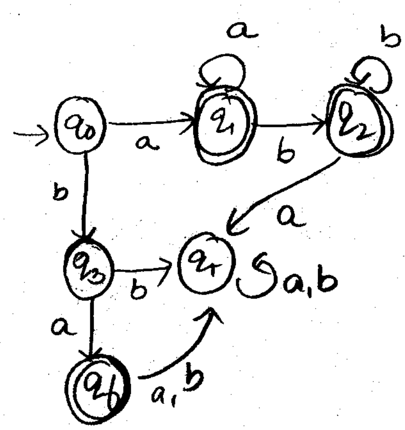
Right Quotient : L_1/L_2

RIGHT QUOTIENT L_1/L_2
→ draw dfa for L_1
→ ∀ nodes apply L_2 to check final state

$L_1 : \{a^n b^m : n \geq 1, m \geq 0\} \cup \{ba\}$

$L_2 : \{b^m : m \geq 1\}$



for L_1/L_2

final states are:

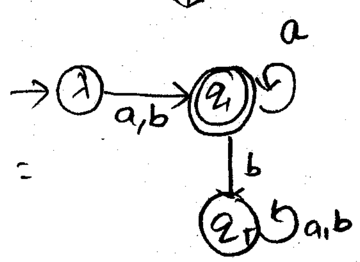
q_1, q_2

Exercises

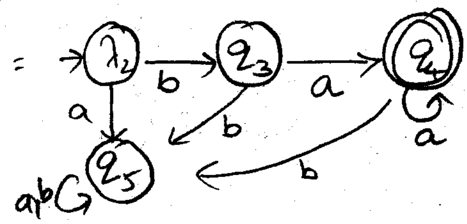
(2) (a) $(a+b)a^* \cap (baa^*)$

nfa = ?

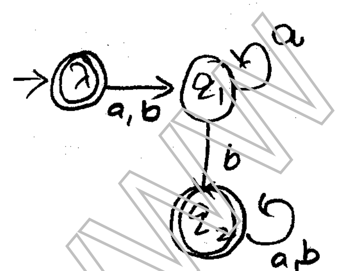
$L_1 =$



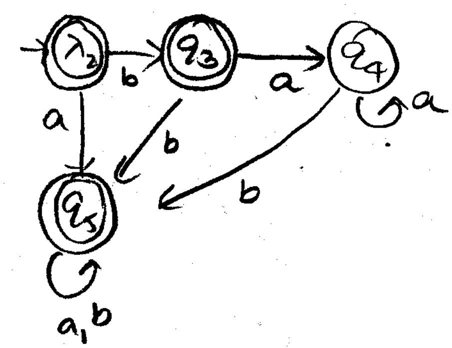
$L_2 =$



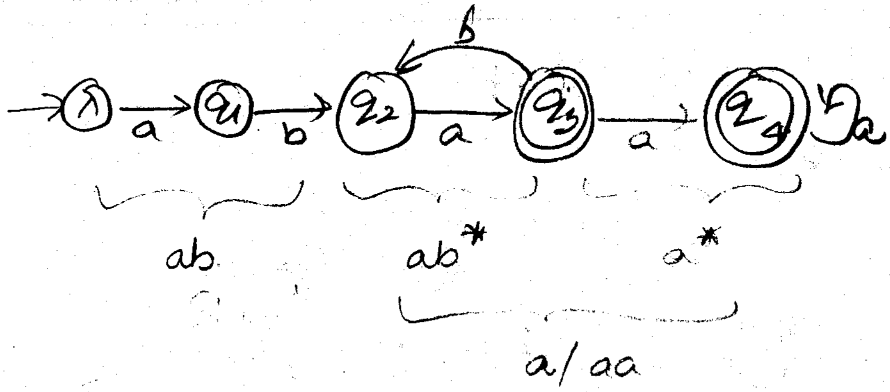
$\overline{L_1} =$



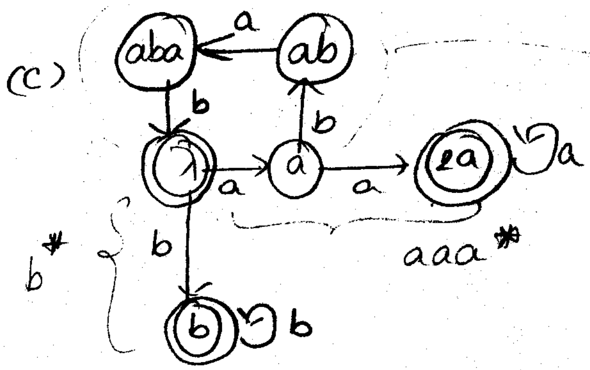
$\overline{L_2} =$



(4) (b) $ab(a+ab)^*(a+aa)$

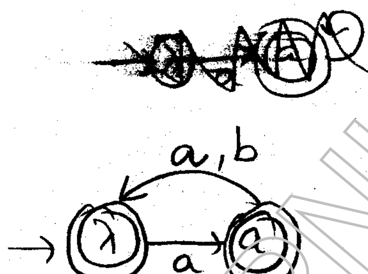


(c)



$(abab)^*$

(d)



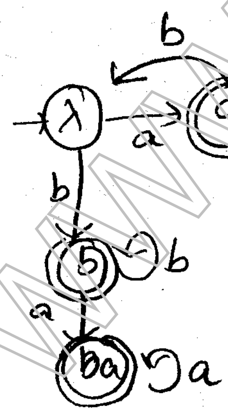
λ, a, aa, ab, abab

$(aa^*b$

(b)

∅

(5) (a)



ab^*a^*

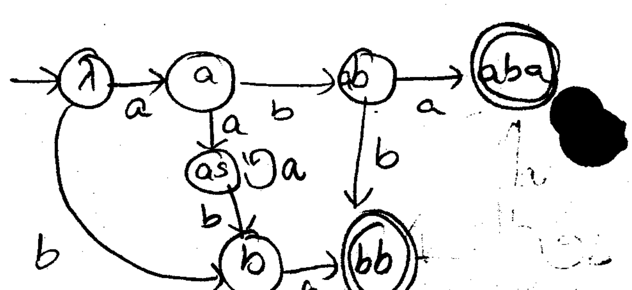
a ✓

aba ✓

abba ✓

ab^*a

abb



(7)

bb ✓

abb ✓

a^*bb

ab^*ba

aba ✓

abba

abbba ✓

a^*bba

Prathima Bhima
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CONTEXT-FREE GRAMMARS

A Grammar $G = (V, T, S, P)$ is CF if all productions in P are of the form

$$\boxed{A \to x}$$

$$\left(x \in (V \cup T)^*, \; A \in V \right)$$

Example 5.1

$G = (\{S\}, \{a,b\}, S, P)$

P:
$S \to aSa$
$S \to bSb$
$S \to \lambda$

$S \to aSa \to abba$

$S \to aSa \to aaSaa \to aabbaa$

$L(G) = \{ww^R : w \in \Sigma^*\}$

Example 5.2

G: P:
$S \to abB$
$A \to aaBb$
$B \to bbAa$
$A \to \lambda$

$S \to abbbAa \to abbba\,(ba)^0$

$\to abbbaaBba \to abbbaabbaba$

$\to abbbaabbaaBbaba \to abbbaabbaabbababa$

$\to abbbaabbaabbAababa \to abbbaabbaabbababa$

$L(G) = \{ab(bbaa)^n\, bba(ba)^m : n, m \geq 0\}$

Example 5.3

ST $L = \{a^n b^m : n \neq m\}$ is context free.

$\left(\begin{array}{l} L: \{a^n b^n : n \geq 0\} \\ S \to aSb / \lambda \end{array} \right)$

[redacted]

$S \to aSb / aA / bB$
$A \to aA / \lambda$
$B \to bB / \lambda$

$S \to AS_1 / BS_1$
$A \to aA / \lambda$
$B \to bB / \lambda$
$S_1 \to aS_1b / \lambda$

Example 5.4

$S \to aSb \,/\, SS \,/\, \lambda$

$L(G) = \{ w : w \in \{a,b\}^*, \; n_a(w) = n_b(w)$ &

$n_a(v) \geq n_b(v)$, v is any prefix of $w \}$

$S \to aSb \to aaSbb \to aabb$

$S \to SS \to aSbaSb \to abab \; \ldots$

Leftmost & Right Most derivations:

$S \to AB$

$A \to aaA$

$A \to \lambda$

$B \to Bb \,/\, \lambda$

$S \to aaABb$

$A \to aaA \,/\, \lambda$

$B \to Bb \,/\, \lambda$

$L(G) = \{ a^{2n} b^m : n, m \geq 0 \}$

Example 5.5

$S \to aAB$

$A \to bBb$

$B \to A \,/\, \lambda$

$S \to aAB \to abBb \to abb$

$S \to aAB \to abBbbBb \to abbbb$

$S \to aAB \to abBbbBb \to abbbbBbb \to abbbbbb$

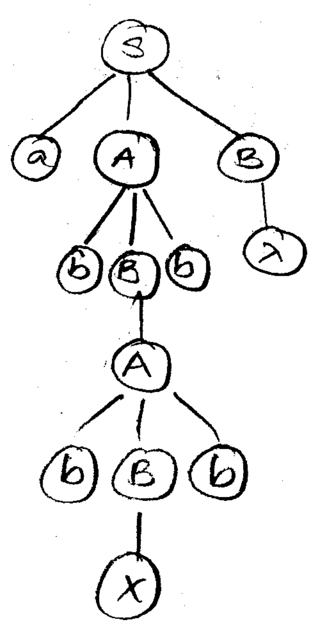
$L(G) = \{ ab^{2n} : n \geq 0 \}$

Example 5.6.

$S \to aAB$

$A \to bBb$

$B \to A \,/\, \lambda$



CHAPTER : 5-1

(CONTEXT FREE GRAMMARS)

Prathima Bhima NOV 3
[redacted]

Def: $G = (V, T, S, P)$

$\boxed{A \to x}$

$x \in (V \cup T)^*$

Eg: 5.1

$G = (\{S\}, \{a,b\}, S, P)$

$S \to aSa$
$S \to bSb$
$S \to \lambda$

$L(G) = \{ww^R : w \in \{a,b\}^*\}$

G is CFG, but not regular.

Eg: 5.2

$S \to abB$
$A \to aaBb$
$B \to bbAa$
$A \to \lambda$

$S \Rightarrow abbbAa \Rightarrow abbba$

$\Rightarrow abbbaabbaba$

$\Rightarrow abbbaabbAaba \Rightarrow abbbaabbaabbababa$

$L(G) = \{ab(bbaa)^n bba(ba)^n : n \geq 0\}$

Eg: 5.3

$L = \{a^n b^m : n \neq m\}$

$n = m$

$S \to aSb / \lambda$

$n > m$

$a^{n+x} b^n$

$S_1 \to AB$
$A \to aA / a$
$B \to aBb / \lambda$

$n < m$, $m > n$

$a^n b^{n+x}$

$S_2 \to BC$
$C \to Cb / b$

$\therefore$

$S \to AB / BC$
$B \to aBb / \lambda$
$A \to aA / a$
$C \to bC / b$

$G = (V, T, S, P)$

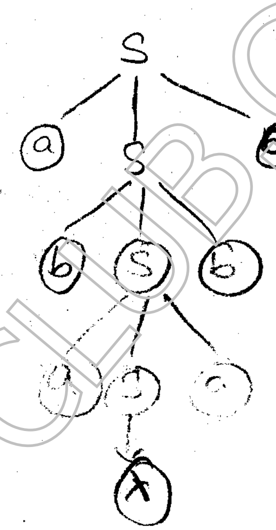
Eg. 5.4 $S \to aSb \,/\, SS \,/\, \lambda$

$\Rightarrow L(G) = \{ w \in \{a,b\}^* : n_a(w) = n_b(w),\ n_a(v) \geq n_b(v)$ where v is prefix of $w \}$

$S \to AB$
$A \to aaA \,/\, \lambda$
$B \to Bb \,/\, \lambda$
$\Big\}\ L = \{a^{2n} b^m : m, n \geq 0\}$

(EXERCISES)

② $S \to aSa$
$S \to bSb$
$S \to \lambda$



⑦ Find CFG for $n \geq 0, m \geq 0$

ⓐ $L = \{a^n b^m : n \leq m+3\}$

$n = m$
$\{ S \to aSb \,/\, \lambda \}$

① $n = m+3 \to$
② $n < m+3 \Rightarrow$ add any no. of b's

$\downarrow$

$S \to aaaA$
$A \to aAb \,/\, B$
$B \to bB \,/\, \lambda$

$n \leq m+3 \Rightarrow n = 0,1,2$

$S \to aA \,/\, aaA \,/\, aaaA \,/\, \lambda$
$A \to aAb \,/\, B$
$B \to bB/\lambda$

$S \to aSb \,/\, aaa$

$n = m+3$
$a^n b^m \Rightarrow a^{m+3} b^m$

$S \to AB$
$A \to aaa$
$B \to aBb \,/\, \lambda$
$\to$ $\cancel{S \to aaaA}$
$\cancel{A \to aAb/}$

$m=0$: $n=0$ λ ✓ $n=1$ a ✓ $n=2$ aa ✓ n

$m=1$: $n=1$ ab ✓ $n=2$ aab ✓ $n=3$ aaab ✓ n= aab

$m=1$ $n=5$ ✗
aaaaab
$aaaA \to aaa$

CH:5·1

EXERCISES.

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[redacted]

7 (b) HW $L = \{a^n b^m : n \neq m-1\}$

$n=0\ m=1$; b ✓
$n=1\ m=2$: abb ✓
$n=1\ m=1$; ab ✗

$n = m-1$
$m = n+1$

$S \to Ab$
$A \to aAb / \lambda$

$n < m-1$
add b's

$S \to Ab$
$A \to aAb / B$
$B \to bB / b$

$n > m-1$
add a's

$S \to Ab$
$A \to aAb / C$
$C \to aC / a$

Test: $n: 0,1,2$; $m=4$

✓ bbbb : $S \to Ab \to Bb \to bbbb$ ✓
abbbb : $S \to Ab \to aAbb \to aBbb \to abbbb$ ✓
aabbbb : $S \to Ab \to aaAbbb \to aabbbb$ ✓
✓ aaabbbb : $S \to Ab \to aaaAbbbb \to aaabbbb$ ✗
aaaabbbb : $S \to aaaaAbbbbb$ ✗

Test $m=3$ $n: 3,4,5,6 \ldots$ ✓

aaabbb : $S \to aaAbbb \to aaabbb$
aaaabbb : $aaAbbb \to aaaCbb \to aaaabbb$ ✓

∴ $n \neq m-1$

⇒

$S \to Ab$
$A \to aAb / B / C$
$B \to bB / b$
$C \to aC / a$

CFG = (   )

Test

HW

7. (c)

$L = \{a^n b^m : n \neq 2m\}$

aaabb

$\boxed{n = 2m}$

$S \to aaSb / \lambda$

$\boxed{n\text{: even}}$

$S \to aaSb / \lambda$

add a's / b's

$S \to aaSb / A / B$

$A \to aA / a$

$B \to bB / b$

$\boxed{n\text{: odd}}$

$S \to aaS / aB$

$B \to bB / \lambda$

$S \to S_1 / S_2$

$S_1 \to aaS_1b / A / B$

$A \to aA / a$

$B \to bB / b$

$S_2 \to aaS_2 / aC$

$C \to bC / \lambda$

$S \to E / O$

$E \to aaEb / \lambda$

and with more a's or more b's.

$O \to aaO / aC$

$C \to bC / \lambda$

$\Rightarrow$

$E \to aaEb / A / B$

$A \to aA / a$

$B \to bB / b$

$\{\lambda \notin L(G)\}$

$\therefore$ CFG: (. . . .)

Test: . . . –

CH # 5.1

(EXERCISES)

Prathima Bhima NOV 5

[redacted]

7 HW d.

$L = \{a^n b^m : 2n \le m \le 3n\}$

$\Rightarrow \quad m = 2n \,/\, m = 3n$

$\therefore \quad S \to aSbb \,/\, aSbbb \,/\, \lambda$

(e) HW $L = \{w \in \{a,b\}^* : n_a(w) \ne n_b(w)\}$

$n_a(w) = n_b(w)$

$S \to$ ~~ss~~ $aSb \,/\, bSa \,/\, \lambda$

add a's or add b's $\Rightarrow$

$S \to SS \,/\, aSb \,/\, bSa \,/\, aS \,/\, bS \,/\, a \,/\, b$

(f) HW $L = \{w \in \{a,b\}^* : n_a(v) \ge n_b(v) : v \text{ is prefix of } w\}$

$S \to SS \,/\, aSb \,/\, \lambda$

(g) HW $L = \{w \in \{a,b\}^*, n_a(w) = 2n_b(w) + 1\}$

$n_a(w) = 2n_b(w)$

$S \to SS \,/\, aaSb \,/\, bSaa \,/\, aSba \,/\, aSab \,/\, abSa \,/\, baSa \,/\, \lambda$

$n_a(w) = n_b(w) + 1$

$S \to SS \,/\, aaSb \,/\, aSba \,/\, aSab \,/\, bSaa \,/\, baSa \,/\, abSa \,/\, a$

Test:

aaab ; $S \to aaSb \to aaab$ ✓

aab : $S \to aSab \to$ ✗

$n \geq 0, m \geq 0, k \geq 0.$

HW 8

(a) $L = \{ a^n b^m c^k : n = m \text{ or } m \leq k \}$

$n = m$, (k)

$S_1 \to AB$

$A \to aAb / \lambda$

$B \to cB / \lambda$

$m \leq k$, (n)

$S \to AB$

$A \to aA / \lambda$

$B \to bBC$

$C \to cC / \lambda$

$S_2 \to DE$

$D \to aD / \lambda$

$E \to bEF$

$F \to cF / \lambda$

$\therefore$ $\boxed{S \to S_1 / S_2}$

$n = m$ (k)

$S_1 \to AB$

$A \to aAb / \lambda$

$B \to cB / \lambda$

$m \leq k$ (n)

$S_2 \to CD$

$C \to aC / \lambda$

$D \to bDc / E$

$E \to cE / \lambda$

$m \leq k$

$m = k$

$X \to bXc$

add c's

$X \to bXc / C$

$C \to cC / \lambda$

HW

(b) $L = \{ a^n b^m c^k : n = m \text{ or } m \neq k \}$

$S \to S_1 / S_2$

$S_1 \to AB$

$A \to aAb / \lambda$

$B \to cB / \lambda$

$m = k$

$X \to bXc / \lambda$

add b's / c's

$X \to bXc / Y / Z$

$Y \to bY / b$

$Z \to cZ / c$

$m \neq k$

$S_2 \to CD$

$C \to aC / \lambda$

$D \to bDc / E / F$

$E \to bE / b$

$F \to cF / c$

HW

(c) $L = \{ a^n b^m c^k : k = n + m \}$

$\underbrace{aa \ldots a}_{n} \underbrace{bb \ldots b}_{m} \underbrace{cc \ldots c}_{n+m}$

for every 'a' add a 'c'

for every 'b' add a 'c'

$S \to aSc / B$

$B \to bBc / \lambda$

$G : (\{S, B\}, \{a, b, c\}, S, P)$

8 (d) $L = \{a^n b^m c^k : n + 2m = k\}$

HW

$\underbrace{aaa \ldots aa}_{n}\underbrace{bb \ldots bb}_{m}\underbrace{cc \ldots c}_{n+2m}$

$\boxed{\text{Every } a \text{ add one } C \quad \text{Every } b \text{ add } 2\ C\text{'s}}$

$S \to aSc / B$
$B \to bBcc / \lambda$

$n:0$
$m:0$
$k:0$

λ ✓
ac
abccc ✓

(e) $L = \{a^n b^m c^k : k = |n-m|\}$

HW

$\underbrace{a \ldots a}_{n}\underbrace{b \ldots b}_{m}\underbrace{c \ldots c}_{\text{excess } a\text{'s or } b\text{'s in the string so far.}}$

$k = n - m$ / $k = m - n$

$\boxed{n = m + k}$ $\boxed{m = n + k}$

$S_1 \to aS_1c /$
$\to a\ b / \lambda$

$S_2 \to aS_2b / B$
$B \to bBc / \lambda$

$S \to S_1 / S_2$

HW

(f) $L = \{w \in \Sigma^* : n_a(w) + n_b(w) \neq n_c(w)\}$

$n + m < k$

$\Rightarrow$ add any no. of c's

$n + m > k$

add any no. of a's or b's or both

$S \to S_1 / S_2$

(8) (9) $L = \{a^n b^m c^k, \ k \neq n+m\}$

HW

$k = n+m$

$S \to aSc / B$

$B \to bBc / \lambda$

$S \to S_1 / S_2$

$n+m < k$

add any no. of c's

$n=m=k$

$S \to aSc / B$

$B \to bBc / \lambda$

$$\boxed{\begin{array}{l} S \to aSc / cS / c / B \\ B \to bSc / \lambda \end{array}}$$

Test abcc : $aSc \to abScc$ ✗

abccc ✓

acc ✓

$n+m > k$

✗ {a, b, aa, ab, ba, bb, abc ....}

$n=m=k$

$S \to aSc / B$

$B \to bBc / \lambda$

add atleast one a or more &

add atleast one b or more

$$\boxed{\begin{array}{l} S_2 \to aS_2c / aS_2 / bS_2 / a / b / B \\ B \to bBc / \lambda \end{array}}$$

Test a : $S \to a$ ✓

aabbcc : $aSc \to aaScc \to aabccc$ ✗

aabcc : $aSc \to aaScc \to aabcc$ ✓

(8) (h) HW $L = \{a^n b^n c^k : k \geq 3\}$

$k \geq 3$

$a^n b^n c^k : n, k \geq 0$

$S \to AB$
$A \to aAb / \lambda$
$B \to cB / \lambda$

$B \to cB / ccc$

minimum 3 c's or more

$\therefore$ $S \to AB$
$A \to aAb / \lambda$
$B \to cB / ccc$

Test

ccc : $S \to AB \to B \to ccc$ ✓

abccc : $S \to abB \to abccc$ ✓

9\. ST $L = \{w \in \{a,b,c\}^* : |w| = 3n_a(w)\}$ is a CFG.

a^n
b^m
c^k

$n + m + k = 3n.$

$\boxed{m + k = 2n}$

$\exists$ for every b an a
$\exists$ for every c an a

(no order $\therefore \Sigma^*$)

$\therefore$ $S \to SS / aSc / B$
$B \to bBc / cBb / \lambda$

Test

sub case

$S \to aSX / bSY / cSZ$
$X \to bc / cb / bb / cc$
$Y \to ac / ca / ab / ba$
$Z \to ab / ba / ac / ca$

$\boxed{m+k = 2n}$

$$\underbrace{aaa}_{n}\ \underbrace{\underbrace{bb\ldots b}_{m}\ \underbrace{ccc}_{k}}_{2n}$$

$S \to ABC$

$A \to$

$m+k = 2n$

$$2\left[n_a(w)\right] = (n_b(w) + n_c(w))$$

Every a has 2 more symbols

↓

Either [b/c]

$\Rightarrow$ $S \to aSX \mid bSY \mid cSZ \mid SS \mid \lambda$

$X \to bb \mid cc \mid bc \mid cb$ ⟷ already 1 a

$Y \to ab \mid ba \mid ac \mid ca$

$Z \to ac \mid ca \mid ab \mid ba$

Test:

$n_a(w) = 1$ $w = abc$: $S \to aSX \to abc$ ✓

$3n_a(w) = 3$ $w = bac$ $S \to bSY \to bac$ ✓

$n_a(w) = 2$ w: aabbbb $S \to SS \to aSX \to$

$3n_a(w) = 6$ $aaSXX \to aabbbb$ ✓

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5.1

11. HW CFG? $L = \{a^n w w^R b^n : w \in \Sigma^*,\ n \geq 1\}$ $\Sigma = \{a, b\}$

$$\left(\begin{array}{l} ww^R \text{ on } \Sigma = \{a,b\} \quad : n \geq 1 \\ S \to aSa \,/\, bSb \,/\, a \,/\, b \end{array} \right)$$

a ✓ aa ✓
b ✓ aba ✓
abba ✓

$S \to aSb \,/\, W$

$W \to aWa \,/\, bWb \,/\, a \,/\, b$

Test

aabab : $S \to aSb \to aaWab \to aabab$ ✓

abab : $S \to aSb \to$ ✗

13. $L = \{a^n b^n : n \geq 0\}$

a) ST L^2 is CFG

b) ST L^k is CFG $\forall\ k \geq 1$

c) ST $\bar{L}$ & L^* are CFG.

L^2: $a^n b^n a^m b^m$

$S \to AA$

$A \to aAb \,/\, \lambda$

L^k : $\underbrace{a^n b^n}_{1} \underbrace{a^m b^m}_{2} \ldots\ldots \underbrace{a^i b^i}_{k}$

$S \to A_1 A_2 \ldots A_{k+1}$

$A \to aAb \,/\, \lambda$

$\bar{L}$: $\Sigma^* - a^n b^n \to$ CFG

$S \to SS \,/\, aSb \,/\, bSa \,/\, \lambda$

$=$ CFG

$=$ CFG

L^* : $\lambda \in L$, L^0 CFG

$L^k \in$ CFG

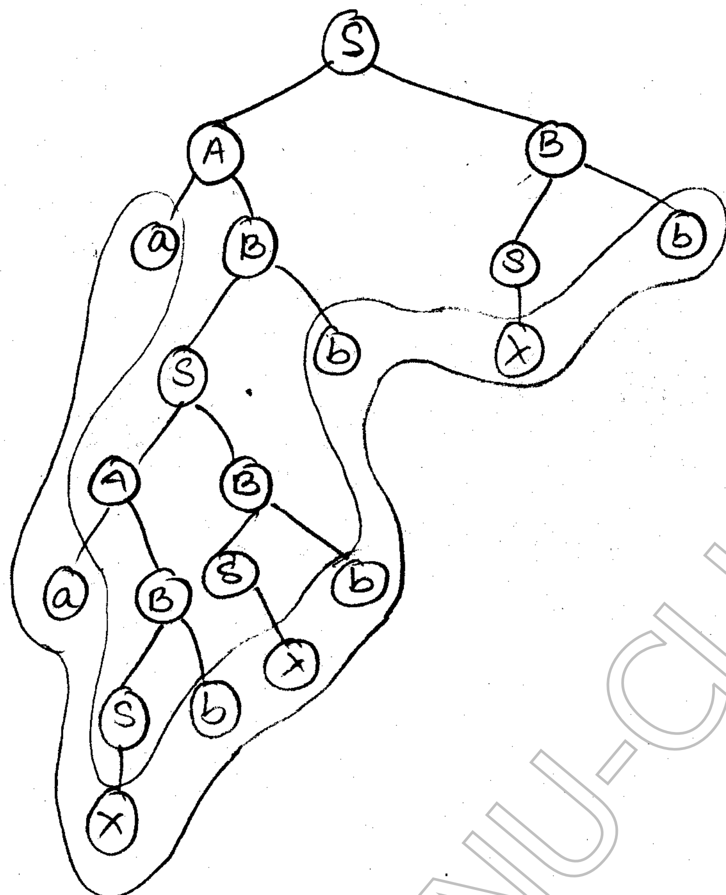
$\therefore L^*$ is CFG

(19) Show derivation Tree for aabbbb

$S \to AB / \lambda$

$A \to aB$

$B \to Sb$



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CH # 5.2
(PARSING & AMBIGUITY)

* **Parsing:** finding a sequence of productions by which $w \in L(G)$ is derived.

Exhaustive search has flaws.

① Tedious

② it is possible that it never terminates for a $w \notin L(G)$

* **SIMPLE GRAMMAR:**

A context free Grammar $G = (V, T, S, P)$ is said to be a simple Grammar or s-grammar if all productions are of the form

→ $\boxed{A \to ax.}$

– $A \in V,\ a \in T,\ x \in V^*$

→ Any pair $\boxed{(A, a)}$ occurs at most once in P.

* A CFG is said to be **ambiguous** if there exists some $w \in L(G)$ that has atleast two distinct derivation trees

*) $S \to aS \,/\, bSS \,/\, c$ ✓ S-Grammar

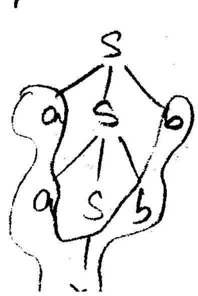
∵ $A \to ax$, (A,a) never repeats

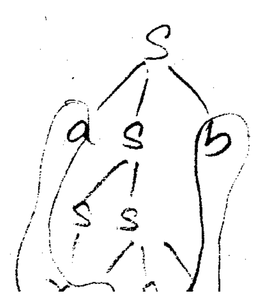
$S \to aS \,/\, bSS \,/\, aSS \,/\, c$ ✗ not S-Grammar

∵ (A,a) repeat though $A \to ax$

* $S \to aSb \,/\, SS \,/\, \lambda$

w: aabb





∴ ambiguous.

→ One way to resolve ambiguity is

① Associate precedence rules ⇒ change semantics

→ Another way is to rewrite the Grammar.

→ If Every Grammar that generates L is ambiguous, then L is called $\boxed{\text{inherently ambiguous.}}$

→

EXERCISES

① find an S-Grammar for $L(aaa^*b+b)$

HW

aaa^*b.

$A \to aX$

(A, a) ×

$S \to aaAb / b$
$A \to aA / \lambda$

$$\boxed{\begin{array}{l} S \to aaAb / b \\ A \to aA / \lambda \end{array}}$$

$S \to aA / b$
$A \to a$
$B \to aB$

$(aaa^*b+b) = (aab + b) + aaa^*b$

0 a^*

min one a

aab

$S \to aX$
$X \to aY$
$Y \to b$

(aaa^*b+b)

$S \to aX / b$
$X \to aY$
$Y \to aY / b$

Test aab: $S \to aX \to aaY \to aab$ ✓

aaab $S \to aX \to aaY \to aaaY \to aaab$ ✓

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CH# 5.2

2. HW find an s-Grammar for $L = \{a^n b^n : n \geq 1\}$

$\{a^n b^n \ \ n \geq 1\}$ $\qquad \lambda \notin L(G)$

$S \to aSb / ab$

$B \to b$

$S \to aSB / aB$

(S, a) ×

$\cancel{S \to aB / b}$

$\cancel{B \to S /}$

$S \to aA$

$A \to b / aAB$

$B \to b$

③ find an s-Grammar for $L = \{a^n b^{n+1} : n \geq 2\}$

$a^n b^{n+1} : n \geq 2$

$S \to aSb / aabbb$

$a^n b^{n+1} : n \geq 0$

$S \to aSb / b$

$n \geq 2$

Substitute $n=2$.

aabbb :

$$\boxed{\begin{array}{l} X \to aY \\ Y \to aZ \\ Z \to bW \\ W \to bV \\ V \to b \end{array}}$$

$\underbrace{- - - -}_{a * n}$ aabbb $\underbrace{- - - -}_{b * n}$

aabbb

$$\boxed{\begin{array}{l} S \to aA \\ A \to aB \\ B \to bX / aBY \\ X \to bY \\ Y \to b \end{array}}$$

$a^n b^n$

Test

aabbb: $S \to aA \to aaB \to aabX \to aabbY \to aabbb$ ✓

aaabbbb: $S \to aA \to aaB \to aaaBY \to aaabXY \to aaabbYY \to aaabbbb$ ✓

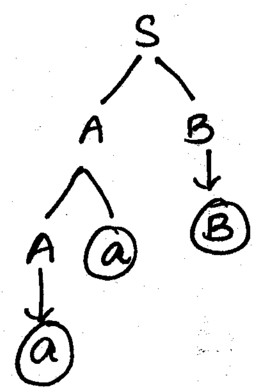
(6) HW Show that the following Grammar is ambiguous:

$S \to AB / aaB$

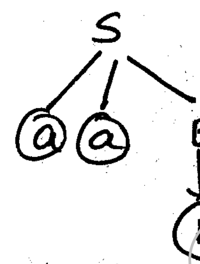
$A \to a / Aa$

$B \to b$

$w = aab$



$w = aab$

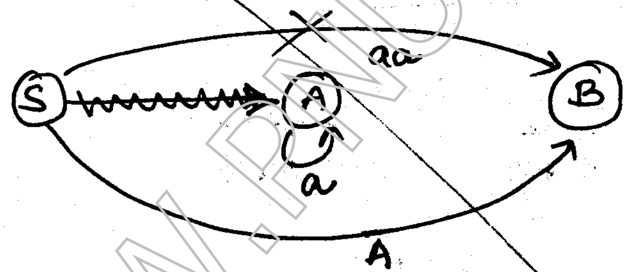


$\exists$ $w = aab$ ST $\exists$ two distinct derivation Trees as above.

$\therefore$ The Grammar is AMBIGUOUS.

(7) HW Construct unambiguous grammar for above Grammar.

$S \to aaB$ is repetitive.



$\therefore$ $S \to AB$

$A \to a / Aa$

$B \to b$

✓ ab

✓ aab

✓ aaab

a^*b

$S \to aA$

$A \to b / aX$

$X \to aX / b$

$\Rightarrow$ a^*b : $\{a^nb : n \geq 1\}$

$\therefore$

~~SAAWAB~~

$S \to aA$

$A \to aA / b$

$S \to aS / b$

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HW ⑧ Give derivation Tree for $(((a+b)*c))+a+b$ using

$E \to T$

$T \to F$

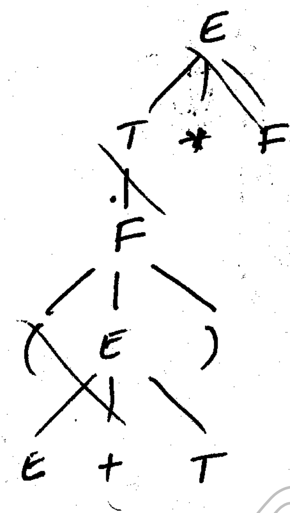
$F \to I$

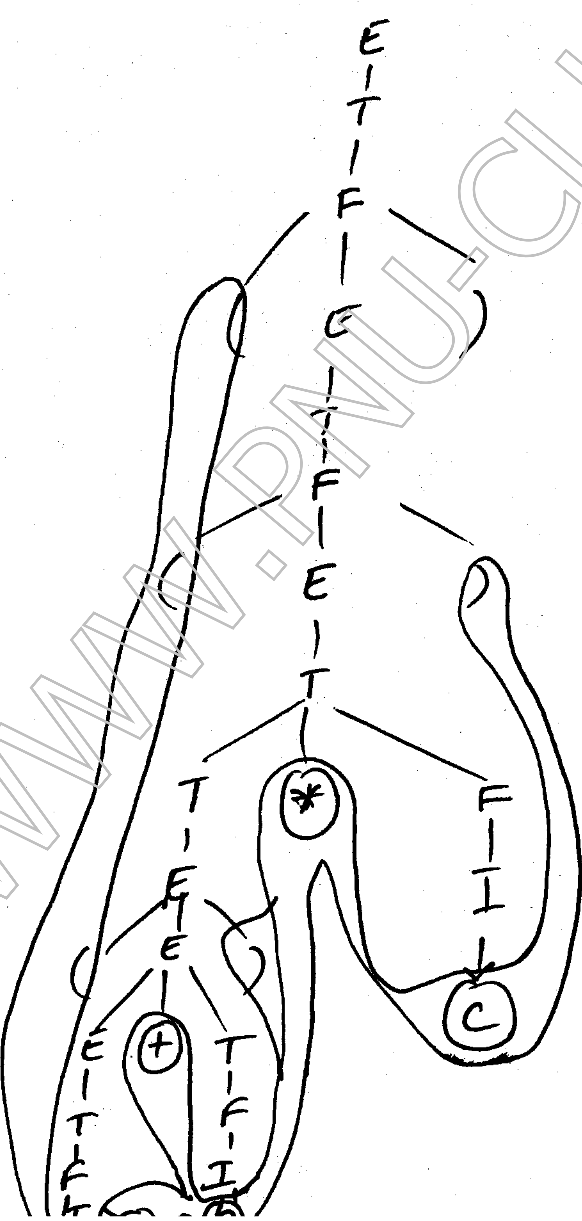
$E \to E+T$

$T \to T*F$

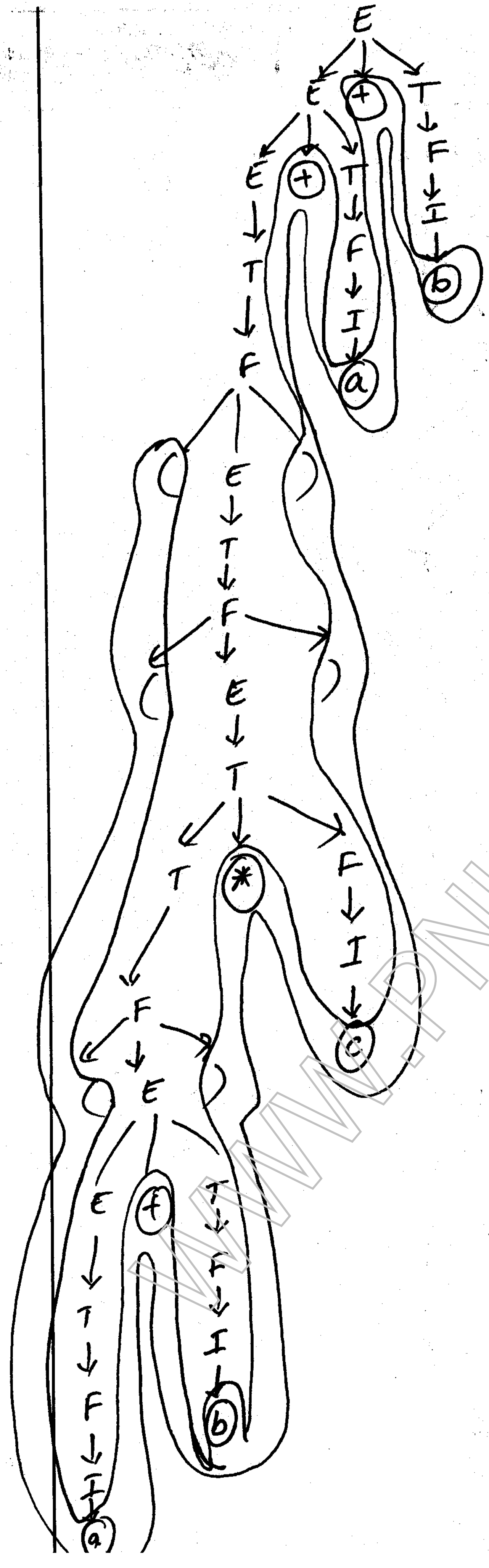
$F \to (E)$

$I \to a/b/c$





$(((a+b)*c))$



$(((a+b)*c))+a+b$

$=$

CH# 5.2

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(10) Give unambiguous grammar equivalent to · set of all regular expressions on $\Sigma = \{a,b\}$

$\{\lambda, a, b, ab, ba, abb \ldots\}$

· SS

$(a+b)^*$

$S \to aS/bS/\lambda$

$S \to aSb/bSa/SS/a/b/\lambda$

ambiguous, strings $\in \{a,b\}^*$

RE:

✓ $S \to SS/aSb/bSa/a/b$

?

$S \to aS/bS/a/b$

$S \to aSX/bSX$

$X \to a/b$

ab ✓

abab ✓

(12) S.T the language $L = \{ww^R : w \in \{a,b\}^*$ is not inherently ambiguous.

all Grammars are ambiguous

ww^R:

$S \to aSa/bSb/a/b/\lambda$

Test ab ✗ abbaabba ✓ aa ✓

aba ✓ $\lambda \notin L(G)$

ww^R

$S \to aSa/bSb/a/b$.

$S \to aSX/bSY$

$X \to a$

$Y \to b$

(1) Eliminate λ

$S \to aSa/bSb/a/b/\lambda$

$S \to aSa/bSb/a/b/aa/bb$

(2) eliminate UNIT-Ps

$S \to aSa/bSb/aa/bb/aaa/bab/aba/bbb$

$S \to aSa/bSb/aaa/aba/bbb/bab/aa/bb$?

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Ch# 6.1

Simplification of CFG & Normal forms

Eg: 6.1

$G = (\{A, B\}, \{a, b, c\}, A, P)$

$A \to a \mid aaA \mid abBc$

$B \to abbA \mid b$

$A \to a \mid aaA \mid ababbAc \mid abbc$

Eg: 6.2

$S \to A$

$A \to aA \mid \lambda$

$\cancel{B \to bA}$

$S \to A$

$A \to aA \mid \lambda$

$S \to A$

$A \to aA \mid a$

$S \to A$

$A \to aA \mid a$

6.3

$S \to aS \mid A \mid C$

$A \to a$

$B \to aa$

$C \to aCb$

$S \to aS \mid a \mid C$

$\cancel{C \to aCb}$

$S \to aS \mid a$

6.4.

$S \to aS_1b$

$S_1 \to aS_1b \mid \lambda$

$S \to aS_1b \mid ab$

$S_1 \to aS_1b \mid ab$

6.5 find CFG without λ-productions

$S \to ABaC$

$\cancel{A \to BC}$

$B \to b \mid \lambda$

$C \to D \mid \lambda$

$D \to d$

① $\lambda \notin L(G)$

② $V_N : \{A, B, C\}$

$S \to ABaC \mid BaC \mid AaC \mid ABa \mid aC \mid Ba \mid Aa \mid a$

$A \to BC \mid B \mid C$

$B \to b$

$C \to D$

$D \to d$

Rules to Eliminate λ-Productions

1. Check that $\lambda \notin L(G)$
2. $V_N = \{ - - - \}$
3. Eliminate all λ-productions
4. make all combinations of nullable variables.

Rules to eliminate UNIT-Productions

STEP #1: find dependency Graph for unit-Productions.

nodes $\longrightarrow$ variable

connections $\exists$ where Unit Production $\exists$.

STEP #2:

$S \overset{*}{\Rightarrow} A$ $A \overset{*}{\Rightarrow} B$

$S \overset{*}{\Rightarrow} B$

$B \overset{*}{\Rightarrow} A$

STEP # 3: Grammar without UNIT-Productions + make Extensions

Eg: 6.6

$S \rightarrow Aa \mid B$

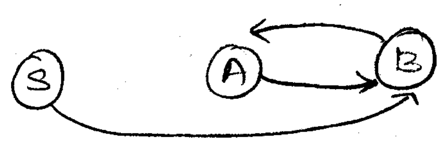
$B \rightarrow A \mid bb$

$A \rightarrow a \mid bc \mid B$

$S \rightarrow B$

$B \rightarrow A$

$A \rightarrow B$



$S \overset{*}{\Rightarrow} A$

$A \overset{*}{\Rightarrow} B$

$B \overset{*}{\Rightarrow} A$

$S \overset{*}{\Rightarrow} B$

$S \rightarrow Aa \;/ a \mid bc \mid bb$

$B \rightarrow bb \;/ a \mid bc$

$A \rightarrow a \mid bc \;/ bb$

[EXERCISES]

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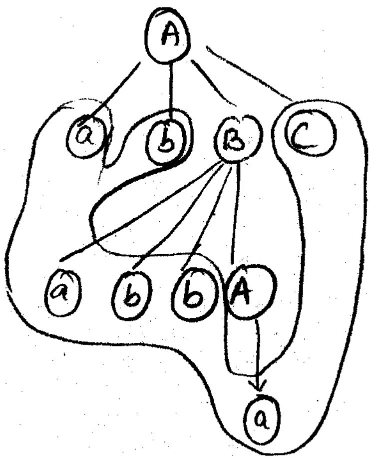
HW ② $A \to a \mid aaA \mid abBc$ $\Rightarrow$ $A \to a \mid aaA \mid$

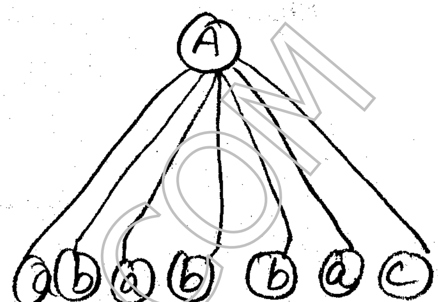
$B \to abbA \mid b$

$ababbac \mid ababbaaAc \mid abbc$.

Derivation tree for $w = ababbac$?



$w = ab\,abbac$



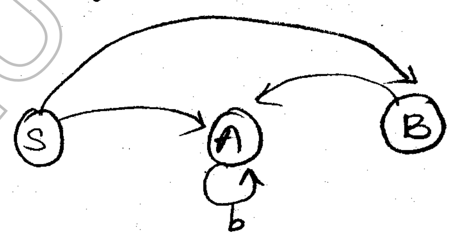
$w = ababbac$

HW ⑤ Eliminate all useless Productions for the Grammar.

$S \to aS \mid AB$

$A \to bA$

$B \to AA$



Substitution:

$S \to aS \mid AAA$

$A \to bA$

~~$B \to AA$~~

$S \to aS \mid AAA$

~~$A \to bA$~~ never ends

$\therefore S \to aS$

never Ends

$\therefore \lambda$.

$L = \{w = a^{\infty} b^{\infty} : w \in \{a,b\}^*\}$

HW

(6) Eliminate Useless Productions from

$S \to a / aA / B / C$

$A \to aB / \lambda$

$B \to Aa$

$C \to cCD$

$\cancel{D \to ddd}$

Substitution:

$S \to a / aA / B / \cancel{cCddd}$

$A \to aB / \lambda$

$\cancel{B \to Aa}$

$\cancel{C \to cCddd}$

$S \to a / aA / Aa$

$A \to aAa / \lambda$

(7) Eliminate λ-Productions from

$S \to AaB / aaB$

$A \to \lambda$

$B \to bbA / \lambda$

① $\lambda \notin L(G)$

② $V_N = \{A, B\}$

$S \to AaB / aaB$

$B \to bbA / \lambda$

$A \to \lambda$

$S \to aB / aaB / a / aa$

$B \to bb$

$S \to aB / aaB / a / aa$

$B \to bb$

Simplified:-

$S \to abb / aabb / a / aa$

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(8) Remove all UNIT-Productions, useless Productions & λ-Productions

$S \to aA / aBB$
$A \to aaA / \lambda$
$B \to bB / bbC$
$C \to B$

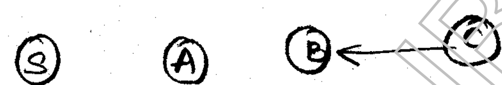
λ-production elimination

(1) $\lambda \notin L(G)$

(2) $V_N = \{A\}$

$S \to aA / aBB / a$
$A \to aaA / aa$
$B \to bB / bbC$
$C \to B$

Unit Production Removal



$C \overset{*}{\Rightarrow} B$

$S \to aA / aBB / a$
$A \to aaA / aa$
~~$B \to bB / bbC$~~

B

$S \to aA / a\cancel{BB} / a$
$A \to aaA / aa$

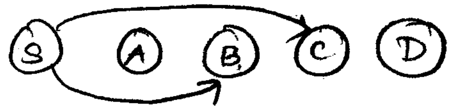
$S \to aA / a$
$A \to aaA / aa$

What does the language generate?

$\{a\} \cup \{a^{2n+1}\}$ $(aa)^* a$

(9) Eliminate UNIT-Productions from (6)

$S \to a / aA / B / C$
$A \to aB / \lambda$
$B \to Aa$
$C \to cCD$
$D \to ddd$



$S \overset{*}{\Rightarrow} B$
$S \overset{*}{\Rightarrow} C$

$S \to a / aA / Aa / cCD$
$A \to aB / \lambda$
$B \to Aa$
$C \to cCD$
$D \to ddd$

Remove λ-Productions

~~S → aS / AB C~~
~~A → a~~
~~B → aa~~
~~C → aCb~~

$S \rightarrow aSb / SS / \lambda$

① $\lambda \in L(G)$

$S \rightarrow aSb / SS / ab$

$sa + \{\lambda\}$ sa

CHAPTER 6-2

CHOMSKY NORMAL FORM:

$A \rightarrow BC$

$A \rightarrow a$

$\lambda \notin L(G)$

→ restrictions on length of Production.

$\{A, B, C\} \in V$

$a \in T$

$S \rightarrow AS / a$
$A \rightarrow SA / b$

$\in CNF$

$S \rightarrow AS / AAS$
$A \rightarrow SA / aa$

$\notin CNF$

Eg 6.8 Convert the Grammar to CNF

$S \rightarrow ABa$
$A \rightarrow aab$
$B \rightarrow AC$

$X \rightarrow a$ $Z \rightarrow c$
$Y \rightarrow b$

$S \rightarrow ABX$ $X \rightarrow a$
$A \rightarrow XXY$ $Y \rightarrow b$
$B \rightarrow AZ$ $Z \rightarrow c$

$S \rightarrow AC$ $D \rightarrow XY$ $Y \rightarrow b$
$C \rightarrow BX$ $B \rightarrow AZ$ $Z \rightarrow c$
$A \rightarrow XD$ $X \rightarrow a$

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GRIEBACH NORMAL FORM:

→ restriction NOT on length of Production
→ but on POSITIONS in which terminals & variables can appea

$$\boxed{A \to ax}$$

$a \in T \quad x \in V^*$

→ looks similar to s-Grammar
→ But no-restriction on (A,a) at Productions.

Eg: 6.9

$S \to AB$
$A \to aA \mid bB \mid b$
$B \to b$

} not GNF $A \to aX$

$S \to aAB \mid bBB \mid bB$
$A \to aA \mid bB \mid b$
$B \to b$

$\in$ GNF.

Eg: 6.10 Convert the Grammar $S \to abSb \mid aa$ into GNF.

$X \to a$
$Y \to b$

$S \to aYSY \mid aX$
$X \to a$
$Y \to b$

for every CFG G, $\lambda \notin L(G)$

$\exists$ Equivalent $\hat{G}$, in GNF.

- EXERCISES -
CH#6.2

(2) Convert to CNF

CNF
$\boxed{\begin{array}{l} A \to BC \\ A \to a \end{array}}$

$S \to aSb \mid ab$

$S \to XSY \mid XY$
$X \to a$
$Y \to b$

$S \to XA \mid XY$
$A \to SY$
$X \to a$
$Y \to b$

CNF

HW (3) Convert to CNF:

$\boxed{\begin{array}{l} \text{CNF:} \\ A \to BC \\ A \to a \end{array}}$

$S \to aSaA \mid A$
$A \to abA \mid b$

Substitution:

$S \to aSaA \mid abA \mid b$
$\cancel{A \to ab}A \mid b$

$S \to aSXA \mid aYA \mid b$
$A \to aYA \mid b$
$X \to a$
$Y \to b$

$S \to XB \mid XC \mid b$
$A \to XC \mid b$
$B \to SD$
$C \to YA$
$D \to XA$

$\cancel{X} \to a$
$Y \to b$

HW (4) Convert to CNF

$\lambda \notin L(G)$

$S \to abAB$
$A \to bAB \mid \lambda$
$B \to BAa \mid A \mid \lambda$

λ Elimination

$V_N : \{A, B\}$

$S \to abAB \mid abA \mid abB$
$A \to bAB \mid bA \mid bB$
$B \to BAa \mid A \mid Ba \mid Aa$

$\boxed{B \to BAa \mid Ba \mid Aa \mid bAB \mid bA \mid bB}$

$S \to \overline{X}\underset{1}{\underline{Y}}AB \mid \overline{X}\underset{2}{\underline{YA}} \mid \overline{X}\underset{1}{\underline{YB}}$
$A \to \underset{3}{\underline{YA}}B \mid \underline{YA} \mid \underline{YB}$
$B \to BAX \mid BX \mid AX \mid \underset{4}{\underline{YA}}B \mid \underline{YA} \mid \underline{YB}$
$X \to a$
$Y \to b$

$S \to XCB \mid XC \mid XD$
$A \to CB \mid YA \mid YB$
$B \to EX \mid BX \mid AX \mid CB \mid YA \mid YB$
$C \to YA$ $\quad E \to BA$ $\quad Y \to b$

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$S \to FB / XC / XD$

$A \to CB / YA / YB$

$B \to EX / BX / AX / CB / YA / YB$

$C \to YA$

$D \to YB$

$E \to BA$

$F \to XC$

$X \to a$

$Y \to b$

CCNF

(5) Convert to CNF:

$\lambda \notin L(G)$

$S \to AB / aB$

$A \to aab / \lambda$

$B \to bbA$

λ-Elimination

$V_N : \{A\}$

$S \to AB / aB / B$

$A \to aab$

$B \to bbA / bb$

Substitution

$S \to AB / aB / bbA / bb$

$A \to aab$

$B \to bbA / bb$

$S \to AB / XB / \underline{YY}B / YY$

$A \to \underline{XX}Y$

$B \to \underline{YY}A / YY$

$X \to a \quad Y \to b$

$S \to AB / XB / CB / YY$

$A \to DY$

$B \to CA / YY$

$C \to YY$

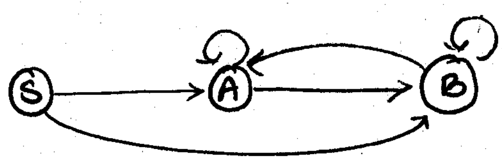
$D \to XX$

HW 7 Draw dependency Graph for

$S \to abAB$

$A \to bAB \mid \lambda$

$B \to BAa \mid A \mid \lambda$



10 Convert to GNF

$S \to aSb \mid bSa \mid a \mid b$

$\boxed{S \to aX}$
GNF

$S \to aSY \mid bSX \mid a \mid b$
$X \to a$
$Y \to b$

11 Convert to GNF

$S \to aSb \mid ab$

$S \to aSY \mid aY$
$Y \to b$

HW 12 Convert to GNF

$S \to ab \mid aS \mid aaS$

$S \to aY \mid aS \mid aXS$
$X \to a$
$Y \to b$

13 Convert to GNF

$S \to ABb \mid a$
$A \to aaA \mid B$
$B \to bAb$

Substitution

$S \to aaAbb \mid BBb \mid a$
$A \to aaA \mid bAb$
$B \to bAb$

$S \to aaABb \mid bAbBb \mid a$
$A \to aaA \mid bAb$
$B \to bAb$

$S \to aXABY \mid bAYBY \mid a$
$A \to aXA \mid bAY$
$B \to bAY$
$X \to a$

(CH # 6.3 SKIP)

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*) Palindrome: CNF = ?

$\lambda \notin L(G)$

$ww^R \rightarrow \cancel{aba}$ $\cancel{bab}$: add a/b

$S \rightarrow aSa \mid bSb \mid \lambda \mid a \mid b$

$S \rightarrow aSa \mid bSb \mid aa \mid bb$

$S \rightarrow XSX \mid YSY \mid XX \mid YY \mid a \mid b$

$X \rightarrow a$

$Y \rightarrow b$

$S \rightarrow XA \mid YB \mid XX \mid YY \mid a \mid b$

$A \rightarrow SX$

$X \rightarrow a$

$Y \rightarrow b$

$B \rightarrow SY$

* $L = \{a^{4n} : n \geq 1\}$ CNF = ?

$S \rightarrow aaaaS \mid aaaa$

$S \rightarrow AAAAS \mid AAAA$

$A \rightarrow a$

$S \rightarrow XXS \mid XX$

$X \rightarrow AA$

$A \rightarrow a$

$S \rightarrow YS \mid XX$

$X \rightarrow AA$

$Y \rightarrow XX$

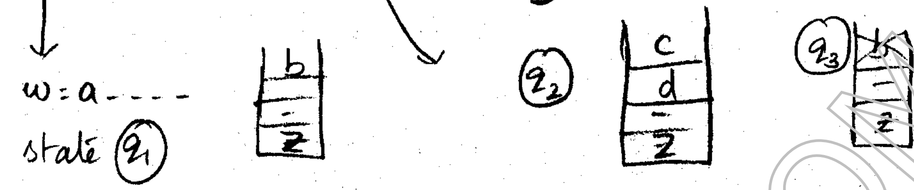
$A \rightarrow a$

Npda: Nondeterministic Pushdown Automata:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$

Eg: 7.1

$$\delta(\underline{q_1}, a, \underline{b}) \rightarrow \{(q_2, cd), (q_3, \lambda)\}$$



Eg: 7.2

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$\Gamma = \{0, 1\}$

$z = 0$

$F = \{q_3\}$

$\delta(q_0, a, 0) = \{(q_1, 10), (q_3, \lambda)\}$

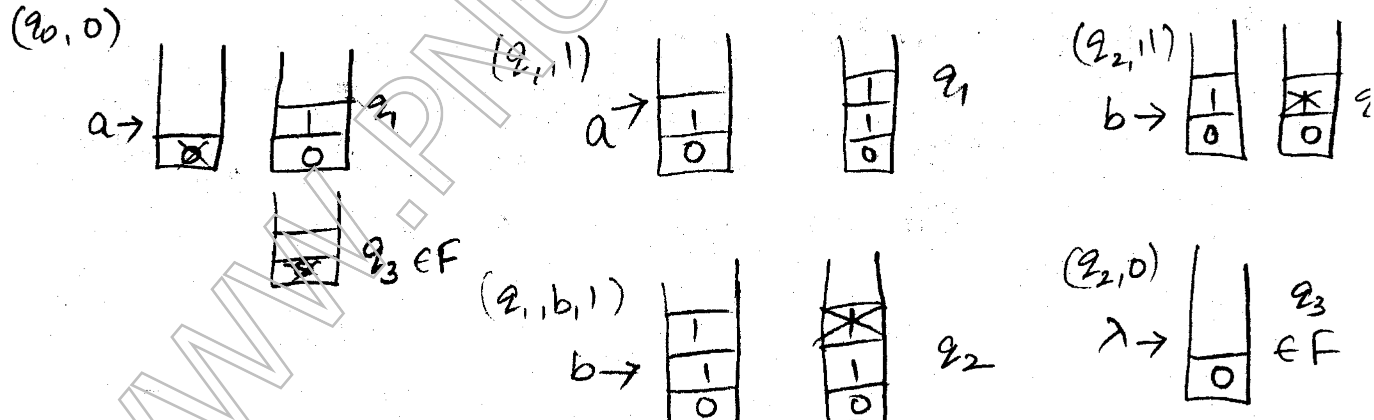
$\delta(q_0, \lambda, 0) = \{(q_3, \lambda)\}$,

$\delta(q_1, a, 1) = \{(q_1, 11)\}$,

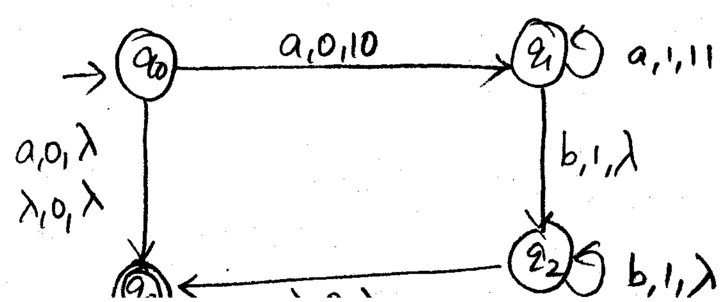
$\delta(q_1, b, 1) = \{(q_2, \lambda)\}$,

$\delta(q_2, b, 1) = \{(q_2, \lambda)\}$

$\delta(q_2, \lambda, 0) = \{(q_3, \lambda)\}$



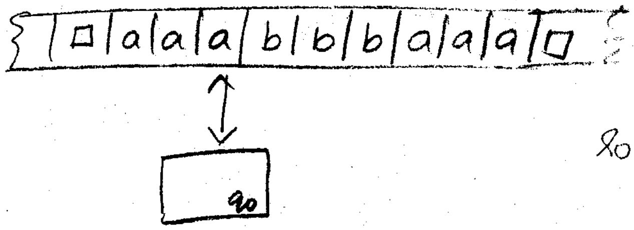
$a \cup \{a^n b^n : n \geq 0\}$



Class

A)

$L = \{a^n b^n a^n \; : n \geq 0\}$



Sol^n:

$a \to x$
$b \to y$
$a \to z$

$\cancel{a}aa\,bb\cancel{b}a\cancel{a}\cancel{a}$

$\cancel{a}a\,b\cancel{b}\cancel{a}\cancel{a}$

$\cancel{a}\cancel{b}\cancel{a}$

$\delta(q_0, a) = (q_1, \square, R)$

$\delta(q_1, a) = (q_1, a, R)$

$\delta(q_1, b) = (q_2, b, R)$

$\delta(q_2, b) = (q_2, b, R)$

$\delta(q_2, a) = (q_3, a, L)$

$\delta(q_3, b) = (q_3, a, R)$

$\delta(q_3, a) = (q_3, a, R)$

$\delta(q_3, \square) = (q_4, \square, L)$

$\delta(q_4, a) = (q_5, \square, L)$

$\delta(q_5, a) = (q_6, \square, L)$

$\delta(q_6, a) = (q_7, a, L)$

$\delta(q_7, a) = (q_7, a, L)$

$\delta(q_7, b) = (q_7, b, L)$

$\delta(q_7, \square) = (q_0, \square, R)$

$\delta(q_6, \square) = (q_f, \square, R)$

$\delta(q_0, \square) = (q_f, \square, R)$

not λ.

$M = (\qquad)$

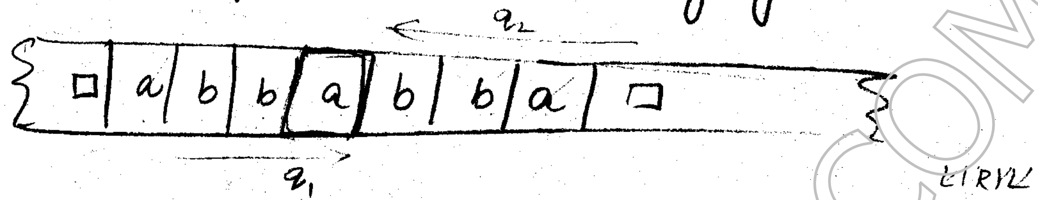
Test $\underline{w = aba} : \in L$

$q_0 aba \vdash \square q_1 ba \vdash b q_2 a \vdash q_3 ba \vdash a q_3 a \vdash$

$aa q_3 \square \vdash a q_4 a \vdash q_5 a \vdash q_6 \square \vdash q_f \square$

accepted

*) Design TM that accepts PALINDROME language.



$\delta(q_0, a) = (q_1, \square, R)$
$\delta(q_0, b) = (q_2, \square, R)$

$\delta(q_1, a) = (q_1, a, R)$ $\quad\delta(q_2, b) = (q_2, b, R)$
$\delta(q_1, b) = (q_1, b, R)$ $\quad\delta(q_2, a) = (q_2, a, R)$
$\delta(q_1, \square) = (q_3, \square, L)$ $\quad\delta(q_2, \square) = (q_5, \square, L)$
$\delta(q_3, a) = (q_4, \square, L)$ $\quad\delta(q_5, b) = (q_4, \square, L)$
$\delta(q_4, a) = (q_4, a, L)$
$\delta(q_4, b) = (q_4, b, L)$
$\delta(q_4, \square) = (q_0, \square, R)$

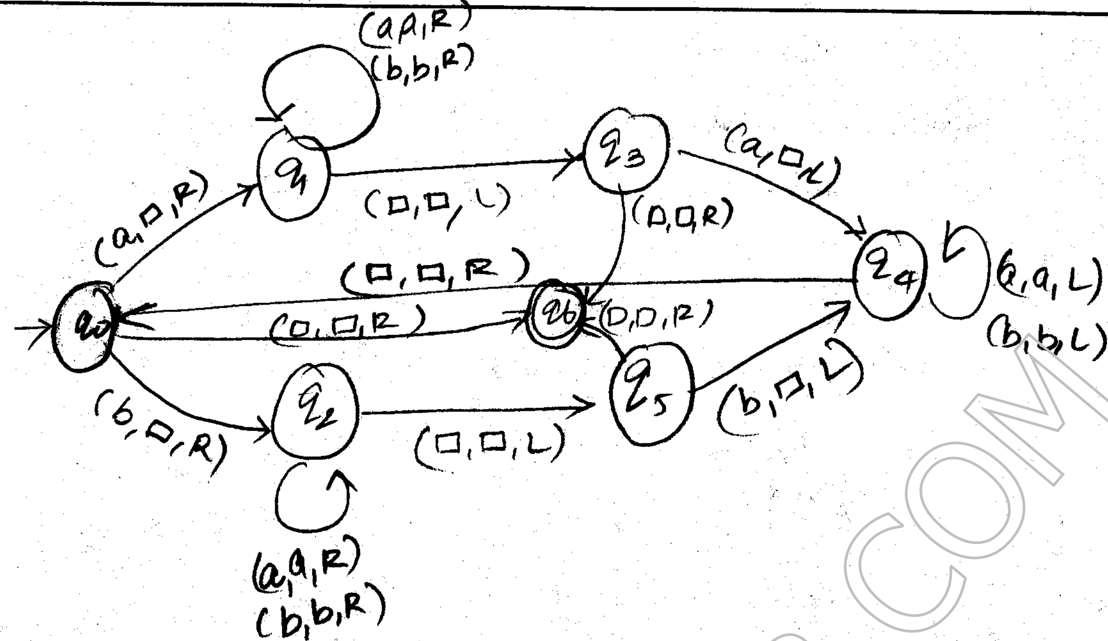
$\delta(q_0, \square) = (q_f, \square, R)$ even string

$\delta(q_5, \square) = (q_f, \square, R)$
$\delta(q_3, \square) = (q_f, \square, R)$ } odd string middle = a/b

M: (_ _)

Test: ababa

insted of expecting another a/b to delete, if no symbol ⇒ accepted



Turing Machine as Transducer:

rejected strings of acceptor = $\bar{L}$

$$\hat{w} = f(w)$$

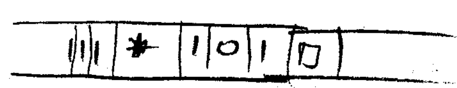
$$\boxed{q_0 w \vdash^{*}_{TM} q_f \hat{w}} \quad (q_f \in F)$$

→ Computable function: ⟺ has TM.

⇒ fn ends @ finite no. of steps
→ whatever the complexity

≃ Algorithm, whatever the complexity.

*) Addition with TM



111
101

use unary NS:

Addition

1	1	0	1	1	1	1	1

- skip till spl. char
- replace it with 1 & move to L
- till □ move to L & del last 1.

$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, \square) = (q_2, \square, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, \square) = (q_f, \square, R)$$

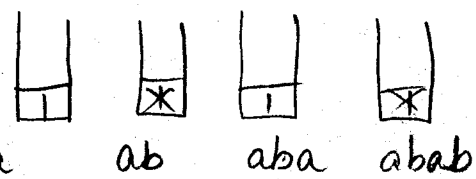
* Every TM has to have R/W @ begning.

CH # 7.1
(PdA)

Prathima Bhima

Eg: 7.4 Construct an npda for the language

$$L = \{ w \in \{a, b\}^* : n_a(w) = n_b(w) \}$$

abab 

$$\delta(q_0, \lambda, z) = \{(q_f, z)\}$$

$M = (\; . \; . \; . \; . \; . \; . \;)$

$$\delta(q_0, a, z) = \{(q_0, 1z)\}$$

$$\delta(q_0, b, z) = \{(q_0, 0z)\}$$

$$\delta(q_0, a, 1) = \{(q_0, 11)\}$$

$$\delta(q_0, b, 0) = \{(q_0, 00)\}$$

$$\delta(q_0, a, 0) = \{(q_0, \lambda)\}$$

$$\delta(q_0, b, 1) = \{(q_0, \lambda)\}$$

<u>Test:</u> $w = abab$

$$(q_0, abab, z) \vdash (q_0, bab, 1z) \vdash (q_0, ab, z) \vdash (q_0, b, 1z) \vdash$$

$$(q_0, \lambda, z) \vdash (q_f, \lambda, z)$$

$\downarrow \in F$ <u>accepted</u>

$w = bbaa$

$$(q_0, bbaa, z) \vdash (q_0, baa, 0z) \vdash (q_0, aa, 00z) \vdash (q_0, a, 0z) \vdash$$

$$(q_0, \lambda, z) \vdash (q_f, \lambda, z)$$

$\downarrow \in F$ accepted.

Eg. 7.5 Construct an npda for $L = \{ww^R : w \in \{a,b\}^+\}$

$$\delta(q_0, a, z) = \{(q_0, az)\}$$
$$\delta(q_0, b, z) = \{(q_0, bz)\}$$
$$\delta(q_0, a, a) = \{(q_0, aa)\}$$
$$\delta(q_0, b, b) = \{(q_0, bb)\}$$
$$\delta(q_0, a, b) = \{(q_0, ab)\}$$
$$\delta(q_0, b, a) = \{(q_0, ba)\}$$

$$\delta(q_0, a, a) = \{(q_1, a)\}$$
$$\delta(q_0, b, b) = \{(q_1, b)\}$$

$$\delta(q_1, a, a) = (q_1, \lambda)$$
$$\delta(q_1, b, b) = (q_1, \lambda)$$

$$\delta(q_1, \lambda, z) = (q_f, z)$$

Test: $\boxed{w = abba}$

$(q_0, abba, z) \vdash (q_0, bba, az) \vdash (q_0, ba, baz) \vdash$

$(q_1, ba, baz) \vdash (q_1, a, az) \vdash (q_1, \lambda, z) \vdash (q_f, \lambda, z)$

$q_f \in F$

accepted

$\boxed{w = abab}$

$(q_0, abab, z) \vdash (q_0, bab, az) \vdash (q_0, ab, baz) \vdash (q_0, b, abaz) \vdash (q_0, \lambda, babaz)$

$\notin F$

$\Rightarrow$ rejected

$$M = (\{q_0, q_1, q_f\}, \{a,b\}, \{a,b,z\}, \delta, z, q_f)$$

CH # 7.1

Prathima Bhima
[redacted]

(3) Construct npda's that accept the following Regular languages

(a) $L_1 = L(aaa^*b)$

$\delta(q_0, a, z) = \{(q_1, az)\}$

$\delta(q_1, a, a) = \{(q_2, aa)\}$

$\delta(q_2, a, a) = \{(q_2, aa)\}$

$\delta(q_2, b, a) = \{(q_f, ba)\}$

$w = aab$

$(q_0, aab, z) \vdash (q_1, ab, az) \vdash (q_2, b, aaz)$

$\vdash (q_f, baaz)$

$\downarrow$

$\in F$

accepted

(b) $L_2 = L(aab^*aba^*)$

$\delta(q_0, a, z) = \{(q_1, az)\}$

$\delta(q_1, a, a) = \{(q_2, aa)\}$

$\delta(q_2, b, a) = \{(q_2, ba)\}$

$\delta(q_2, b, b) = \{(q_2, bb)\}$

$\delta(q_2, a, a) = \{(q_3, aa)\}$

$\delta(q_2, a, b) = \{(q_3, ab)\}$

$\delta(q_3, b, a) = \{(q_f, ba)\}$

$\delta(q_f, a, b) = \{(q_f, ab)\}$

$\delta(q_f, a, a) = \{(q_f, aa)\}$

$\delta(q_f, \lambda, b) = \{(q_f, b)\}$

$\delta(q_f, \lambda, a) = \{(q_f, a)\}$

$\boxed{w = aaab}$

$(q_0, aaab, z) \vdash (q_1, aab, az) \vdash (q_2, ab, aa$

$\vdash (q_3, b, aaa) \vdash (q_f, \lambda, baaa) \vdash$

$(q_f, \lambda, baaa)$

$\downarrow$

$\in f$ accepted

$M(\ldots)$

(c) $L_1 \cup L_2$: $(aaa^*b) \cup (aab^*aba^*)$

$\delta(q_0, a, z) = \{(q_1, az)\}$

$\delta(q_1, a, a) = \{(q_2, aa)\}$

$\delta(q_2, a, a) = \{(q_3, aa)\}$ a^+ aaa^*

$\delta(q_2, b, a) = \{(q_4, ba), (q_f, ba)\}$ $\boxed{aab \text{ final}}$, $\boxed{aaa^*b \text{ final}}$

$\delta(q_3, b, a) = \{(q_4, ba)\}$

$\delta(q_3, b, b) = \{(q_3, bb)\}$

$\delta(q_3, a, b) = \{(q_4, ab)\}$

$\delta(q_4, b, a) = \{(q_5, ba)\}$ $\boxed{aab^*ab \text{ final}}$

$\delta(q_5, a, a) = \{(q_f, aa)\}$ $\boxed{aab^*aba^+ \text{ final}}$

$\delta(q_f, \lambda, z) = \{(q_f, \lambda)\}$

Test:

aabab:

store

$\delta(q_0, aabab, z) \vdash \delta(q_1, abab, az) \vdash \delta(q_2, bab, aaz) \vdash \cancel{\boxed{\delta(q_f, ab, baaz)}}$

$\vdash \delta(q_4, ab, baaaz) \vdash$

npda $\Longleftrightarrow$ CFG

CFG $\rightarrow$ npda

$\boxed{\text{CFG} \rightarrow \text{GNF} \rightarrow \text{npda}}$

$\downarrow$

$\boxed{A \rightarrow aX}$

$\delta(q_0, \lambda, z) = (q, Sz)$

$\boxed{\delta(q_1, a, A) = (q_1, X)}$

$\delta(q_1, \lambda, z) = (q_f, \lambda)$

76. Eg: $S \rightarrow aSbb \,/\, a$

$$\left.\begin{array}{l} S \rightarrow aSYY \,/\, a \\ Y \rightarrow b \end{array}\right\} \text{GNF}$$

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, SYY), (q, \lambda)\}$

$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

$\delta(q_1, b, Y) = \{(q_1, \lambda)\}$

$M = (\ldots)$

Test

Q: 7.7. $S \to aA$ npda = ?

$A \to aABC \mid bB \mid a$

$B \to b$

$C \to c$

$\to$ $\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, A)\}$

$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$

$\delta(q_1, b, A) = \{(q_1, B)\}$

$\delta(q_1, b, B) = \{(q_1, \lambda)\}$

$\delta(q_1, c, C) = \{(q_1, \lambda)\}$

$\to$ $\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

EXERCISES

1
2
5

$S \to aABB \mid aAA$ npda = ?

$A \to aBB \mid a$

$B \to bBB \mid A$.

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, ABB), (q_1, AA)\}$

$\delta(q_1, a, A) = \{(q_1, BB), (q_1, \lambda)\}$

$\delta(q_1, b, B) = \{(q_1, BB), (q_1, A)\}$

$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

CH: 7 #2

Prathima Bhima
[redacted]

(a) find npda with 2 states for $L = \{a^n b^{n+1} : n \geq 0\}$

$S \to aSb \mid b$

GNF: $S \to aSY \mid b$

$Y \to b$

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, SY)\}$

$\delta(q_1, b, Y) = \{(q_1, \lambda)\}$

$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

$\delta(q_1, b, S) = \{(q_1, \lambda)\}$

$\delta(q_0, \lambda, z) = \{(q_0, Sz_1)\}$

$\delta(q_0, a, S) = \{(q_0, SY)\}$

$\delta(q_0, b, S) = \{(q_0, \lambda)\}$

$\delta(q_0, b, Y) = \{(q_0, \lambda)\}$

$\delta(q_0, \lambda, z_1) = \{(q_f, \lambda)\}$

(b) find npda with 2 states that accepts $L = \{a^n b^{2n} : n \geq 1\}$

$S \to aSbb \mid \lambda$

GNF: $S \to aSbb \mid abb$

$S \to aSBB \mid aBB$

$B \to b$

$\delta(q_0, \lambda, z) = \{(q_0, Sz_1)\}$

$\delta(q_0, a, S) = \{(q_0, SBB), (q_0, BB)\}$

$\delta(q_0, b, B) = \{(q_0, \lambda)\}$

$\delta(q_0, \lambda, z_1) = \{(q_f, \lambda)\}$

CH # 7.3

$\boxed{\text{Dpda : DCFL}}$

Dpda

① $\delta(q, a, b)$ contains atmost one element

② if $\delta(q, \lambda, b) \neq \emptyset$

then $\delta(q, c, b) = \emptyset \quad \forall\ c \in \Sigma$

Eg: 7.10 $L = \{a^n b^n : n \geq 0\}$ is DCFL.

Dpda = ?

$$\delta(q_0, a, 0) = \{(q_1, 10)\}$$
$$\delta(q_1, a, 1) = \{(q_1, 11)\}$$
$$\delta(q_1, b, 1) = \{(q_2, \lambda)\}$$
$$\delta(q_2, b, 1) = \{(q_2, \lambda)\}$$
$$\delta(q_2, \lambda, 0) = \{(q_0, \lambda)\}$$

$q_0 \in F$

*) $L = \{a^n b^n : n \geq 0\} \cup \{a\}$

dpda = ?

$\{q_0, q_f, q_2\} \in F$

$$\delta(q_0, a, z) = (q_f, az)$$
$$\delta(q_f, a, a) = (q_1, aa)$$
$$\delta(q_f, b, a) = (q_f, \lambda)$$
$$\delta(q_f, \lambda, z) = (q_f, \lambda)$$
$$\delta(q_1, a, a) = (q_1, aa)$$
$$\delta(q_1, b, a) = (q_2, \lambda)$$
$$\delta(q_2, b, a) = (q_2, \lambda)$$

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[redacted]

CH # 7.3
Dpda EXERCISES

① ST. $L=\{a^n b^{2n} : n \geq 0\}$ is a DCFL. abb.

$\delta(q_0, \lambda, z) = (q_f, \lambda)$
$\delta(q_0, a, z) = (q_1, 11z)$
$\delta(q_1, a, 1) = (q_1, 111)$
$\delta(q_1, b, 1) = (q_1, \lambda)$
$\delta(q_1, \lambda, z) = (q_f, \lambda)$

dpda
∴ DCFL

Test abb: accepted

$\delta(q_0, abb, z) \vdash \delta(q_1, bb, 11z) \vdash \delta(q_1, b, 1z) \vdash \delta(q_1, \lambda, z) \vdash (q_f, \lambda)$

aabbbb: accepted

$\delta(q_0, aabb\overset{bb}{}, z) \vdash \delta(q_1, abb\overset{bb}{}, 11z) \vdash \delta(q_1, bb\overset{bb}{}, 1111z) \vdash \delta(q_1, bbb, \ldots$

③ Is $L=\{a^n b^n : n \geq 1\} \cup \{b\}$ DCFL?

$\delta(q_0, a, z) = (q_1, 1z)$
$\delta(q_0, b, z) = (q_f, \lambda)$
$\delta(q_1, a, 1) = (q_1, 11)$
$\delta(q_1, b, 1) = (q_1, \lambda)$
$\delta(q_1, \lambda, z) = (q_f, \lambda)$

∴ dpda
∴ DCFL ✓

5

8 $L = \{a^n b^m : n = m \text{ or } n = m + 2\}$ is DCFL ?

$\{a^n b^n\} \cup \{a^{n+2} b^n\}$

9 wcw^R

$\downarrow$

start matching.

CH # 8.1
Properties of CFL

Eg 8.1

ST $L = \{a^n b^n c^n : n \geq 0\}$ is not context free.

1. $w = a^m b^m c^m \in L$

a . . . ab . . . bc c

(4) (5)

(1) (2) (3)

2.1 $v = a$
$y = a$
$w_i = a^{m+2i-2} b^m c^m$

$i > 1 \Rightarrow m+2i-2 > m$

$w_i \notin L$

$\because n_a(w_i) \neq n_b(w_i)$
$\neq n_c(w_i)$

$\boxed{w_i \notin L}$

2.2 $v = b$, $y = b$ 2.3 $v = c$, $y = c$ — similar cases

2.4. $v = a$
$y = b$

$w_i = a^{m+i-1} b^{m+i-1} c^m$

$i > 1 \Rightarrow m+i-1 > m$

$\Rightarrow n_a(w_i) \neq n_c(w_i)$
$n_b(w_i) \neq n_c(w_i)$

$\boxed{w_i \notin L}$

2.5. $v = b$, $y = c$ — Similar case

∴ as Pumping Lemma fails, L is not a CFL.

Eg: # 8.2

$L = \{ww : w \in \{a,b\}^*\}$ is L CFL?

a . . ab . . ba . . ab . . . b

(1) (2) (3) (4)

(5) (6) (7)

1. $w = a^n b^n a^m b^m$

2.1 $w_i = a^{n+2i-2} b^n a^m b^m$

$i > 1 \Rightarrow n+2i-2 > m$

$w_i \notin L$

$v = a$, $y = a$ {first w}

2.2. $v = b, y = b$
2.3. $v = a, y = a$
2.4 $v = b, y = b$
similar cases

2.5 $v = a, y = b$

$w_i = a^{n+i-1} b^{n+i-1} a^m b^m$

$i > 1 \Rightarrow n_a(\text{first } w) > n_a(\text{second } w)$

$\therefore w_i \notin L$

2.6 $v = b, y = a$
2.7 $v = a, y = b$
similar case

L is not CFL as PL fails.

Eg: 8.3 ST. $L = \{ a^{n!} : n \geq 0 \}$ is not context free.

1. $w = a^{m!} \in L$

2. $v = a^k$
$y = a^l$
$w_i = a^{[m-(k+l)+2]!}$

$k+l < m$: $m-(k+l) > 0$

$\therefore m-(k+l) > m!$

$\therefore$ not CFL.

Eg: 8.4 ST $L = \{ a^n b^j : n = j^2 \}$ is not CFL.

$$\overset{①}{aa\ldots} \;\; \overset{③}{\overbrace{aab\ldots}} \;\; \overset{②}{\ldots b}$$

1. $w = a^{m^2} b^m \in L$

2.1 $v = a$
$y = a$
$w_i = a^{m^2+2i-1} b^m$
$i > 1 \Rightarrow m^2+2i-1 > m^2$
$\therefore w_i \notin L$
$(\simeq$ 2.2 $v = b, y = b)$

2.3 $v = a$
$y = b$
$w_i = a^{m^2+i-1} b^{m+i-1}$
$i = 0$: $m^2 - 1 \neq (m-1)^2$
$w_i \notin L$

$\therefore L$ is not CFL

4) $L = \{ a^n b^j a^j b^n : n, j > 0 \}$ is CFG or not?

$\delta(q_0, a, z) = \{(q_0, az)\}$
$\delta(q_0, a, a) = \{(q_0, aa)\}$
$\delta(q_0, b, a) = \{(q_1, ba)\}$
$\delta(q_1, b, b) = \{(q_1, bb)\}$
$\delta(q_1, a, b) = \{(q_2, \lambda)\}$
$\delta(q_2, a, b) = \{(q_2, \lambda)\}$
$\delta(q_2, b, a) = \{(q_3, \lambda)\}$
$\delta(q_3, b, a) = \{(q_3, \lambda)\}$
$\delta(q_3, \lambda, z) = \{(q_f, \lambda)\}$

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[redacted]

CH#8.1

$L = \{ a^n b^n c^j : n \le j \}$

1. $w = a^m b^m c^{m+1} \in L$

$$(\underset{①}{a \ldots} \; \overset{④}{a b} \underset{②}{\ldots} \overset{⑤}{b c} \underset{③}{\ldots c})$$

2.1 $v = a$, $y = a$

$w_i = a^{m+2i-2} b^m c^{m+1}$

$i > 1$ $m+2i-2 > m$

$\Rightarrow n_a(w) \ne n_b(w)$

$w_i \notin L$

2.2 similar cases

2.3 $v = c$, $y = c$

$w_i = a^m b^m c^{m+2i-2+1}$

$i = 0$

$m-1 < m$

$w_i \notin L$

2.4 $v = a$, $y = b$

$w_i = a^{m+i-1} b^{m+i-1} c^{m+1}$

$i > 2 \Rightarrow m+i-1 > m+1$

$\therefore w_i \notin L$

$n_c(w) \not> n_a(w)$

$\not> n_b(w)$

$\cong$ 2.5

$\therefore L \notin CFL$

EXERCISES

(2) ST $L = \{ a^n : n \text{ is a prime no.} \}$ is not CFL.

1. $w = a^m$ m is prime $: \in L$

2. $a \ldots\ldots\ldots a$

$v = a$

$y = a$

$w_i = a^{m+2i-2}$

$i = 0$ $m+2i-2 = m-2$

$m-2$ is not prime

$i > 0$ $m+2i-2$

$i < 0$ $m+2i-2$ } not necessarily prime

$\therefore$ PL fails $\Rightarrow$ NOT CFL

(5) Is $L = \{a^n b^m : n = 2^m\}$ CFL?

1. $w = a^{2^m} b^m \in L$

2. aaa . . abb . . . b

(1) (2) (3)

2.1 $v = a$, $y = a$

$w_i = a^{2^m + 2i - 2} b^m$

$\boxed{2^m + 2i - 2}$

$i = 0 : \quad 2^m - 2 \neq 2^m$

ie. $Pow(a) \neq 2^m$

$\therefore w_i \notin L$

2.2 Similar for $v = b$, $y = b$

2.3 $v = a$, $y = b$

$w_i = a^{2^m + i - 1} b^{m + i - 1}$

$i = 0 : \quad a^{2^m - 1} b^{m-1}.$

$2^{m-1} = 2^m - 2 = 2^m - 1 - 1$

$\boxed{2^{m-1} > 2^m - 1}$

ie. $Pow(a) \neq 2^m$

$\therefore w \notin L$

ST not CFL:

(7) a) $L = \{a^n b^j : n \leq j^2\}$

$w = a^{j^2} b^j$

not CFL

b) $L = \{a^n b^j : n \geq (j-1)^3\}$

$w = a^{(j-1)^3} b^j$.

not CFG

(f) $n_a(w) = n_b(w) < n_c(w)$

$a^n b^{n+1} c^{n+2}$

(8) CFL or not?

(a) $L = \{a^n w w^R a^n : n \geq 0, \; w \in \{ab\}^*\}$

ww^R : CFL
a^n : CFL
} CFL closed under concatenation $\Rightarrow \boxed{CFL}$

$\delta(q_2, a, b) = \{(q_3, \lambda)\}$
$\delta(q_3, a, a) = \{(q_3, \lambda)\}$
$\delta(q_3, \lambda, z) = \{(q_4, \lambda)\}$

$\delta(q_0, a, z) = \{(q_0, az), (q_1, az)\}$
$\delta(q_0, a, a) = \{(q_0, aa), (q_1, aa)\}$
$\delta(q_1, a, a) = \{(q_1, aa)\}$
$\delta(q_1, b, a) = \{(q_1, ba)\}$
$\delta(q_1, a, b) = \{(q_1, ab), (q_2, \lambda)\}$
$\delta(q_1, b, b) = \{(q_1, bb), (q_2, \lambda)\}$
$\delta(q_2, a, a) = \{(q_2, \lambda)\}$
$\delta(q_2, b, b) = \{(q_2, \lambda)\}$

CH # 8.1

8 b.

$L = \{a^n b^j a^n b^j : n \geq 0,\ j \geq 0\}$

not CFG

1. $w = a^m b^k a^m b^k \quad \in L$

5 6 7

aa . . . . ab . . . ba . . ab . . . b

1 2 3 4

2.1 $v = a$
$y = a$

$\boxed{2.2}$ $\boxed{2.3}$ $\boxed{2.4}$

$w_i = a^{m+2i-2} b^k a^m b^k$

$i > 0 \ : \ m+2i-2 > m$

$w_i \notin L$

2.5 $v = a$
$y = b$

$\boxed{2.6}$ $\boxed{2.7}$

$w_i = a^{m+i-1} b^{m+i-1} a^m b^m$

$i = 0 \quad m-1 \neq m$

$w_i \notin L$

PL fails $\Rightarrow$ NOT CFL

c $L = \{a^n b^j a^j b^n : n \geq 0,\ j \geq 0\}$

$\boxed{a^n b^j}$

$\delta(q_0, a, z) = \{(q_0, az)\}$
$\delta(q_0, a, a) = \{(q_0, aa)\}$
$\delta(q_0, b, a) = \{(q_1, ba)\}$

$\delta(q_0, b, z) = \delta(q_1, bz)$

$\boxed{a^0 b^j}$

$\delta(q_1, b, b) = \{(q_2, bb)\ (q_3, \lambda)\}$
$\delta(q_2, a, b) = \{(q_1, \lambda)\}$
$\delta(q_2, b, b) = \{(q_3, \lambda)\}$

$\boxed{b^j a^j}$

$\boxed{b^0 a^0}$

$\delta(q_3, b, b) = \{(q_3, \lambda)\}$
$\delta(q_3, \lambda, z) = \{(q_f, \lambda)\}$

$\boxed{a^n \times b^n}$

$\delta(q_0, \lambda, z) = \{(q_f, \lambda)\} \longrightarrow \lambda \in L(G)$

$\therefore$ CFL

(d) $L = \{a^n b^j a^k b^l : n+j < k+l\}$

1. $w = a^n b^n a^m b^m$ $\quad n < m$

$$a \ldots \underbrace{ab}_{⑤} \ldots \underbrace{ba \ldots ab}_{⑥} \ldots b$$

① ② ③ ④

2.1 $v = a$
$y = a$
$w_i = a^{n+2i-2} b^n a^m b^m$

$i > 0 \quad n + 2i > m$

but $n < m$

$\therefore w_i \notin L$

2.2
2.3
2.4

2.5 $v = a$
$y = b$

(2.6)

$w_i = a^{n+i-1} b^{n+i-1} a^m b^m$

POW(LHS) POW(RHS)

$2n+2i-2 \quad : \quad 2m$

$\boxed{n+i-1} \quad : \quad \boxed{m}$

$n < m$

$i > 0 \quad n+i > m$

$w_i \notin L$

As PL fails, NOT CFL

(e) $L = \{a^n b^j a^k b^l : n \leq k,\ j \leq l\}$

NOT CFL

(f) $L = \{a^n b^n c^j : n \leq j\}$

NOT CFL

(g) $L = \{w \in \{a,b,c\}^* : n_a(w) = n_b(w) = 2n_c(w)\}$

NOT CFL.

CH # 8.2

CFL closed under $\cup$ union
$\cdot$ concatenation
$\rightarrow$ $*$ Star Closure.
$\rightarrow$

$\boxed{RL \cap CFL = CFL}$

NOT closed under $\rightarrow$ Intersection $\cap$
$\rightarrow$ Complement $\bar{A}$:

Eg: 8.7 ST $L = \{a^n b^n : n \geq 0, n \neq 100\}$ is CFL.

$L_2 = \{a^n b^n : n \geq 0\}$

$L_1 = \{a^n b^n : n = 100\}$

$L_1 = \{a^{100} b^{100}\}$ $\longrightarrow$ Regular

Regular languages are closed under complement

so $\bar{L_1}$ is also regular.

$$L = \underbrace{L_2}_{CFL} \cap \underbrace{\bar{L_1}}_{RL} = \{a^n b^n \; n \neq 100, n \geq 0\}$$

$\therefore$ L is a CFL.

Eg: 8.8 ST $L = \{\{a, b, c\}^* : n_a(w) = n_b(w) = n_c(w)\}$ is not CFL.

PL fails: also: $L_1 = (a^* b^* c^*)$ $\rightarrow$ Regular

we know $L_2 \{a^n b^n c^n\}$ is NOT CFL.

$L \cap L_1 = L_2$
? Regular NOT CFG

Case (i) L is CFL $\Rightarrow$ L_2 should be CFL, but is not $\Rightarrow$ $\boxed{L \text{ is NOT CFL}}$

Case (ii) $\boxed{L \text{ is not CFL}}$ $\Rightarrow$ L_2 is not CFL, true :

Is Empty / Is NOT Empty

$\boxed{\begin{array}{l} N \to t \\ N \to X \end{array}}$

Eg:

$S \to XY$	$S \to XY$	$S \to aabb$
$X \to AX$	$\cancel{X \to aX}$	
$X \to AA$	$X \to aa$	L(a) NOT empty
$A \to a$	$\cancel{Y \to bY}$	
$Y \to BY$	$Y \to bb$	
$Y \to BB$		
$B \to b$		

Eg:

$S \to XY$	$S \to XY$	$S \to Xbb$
$X \to AX$	$X \to aX$	$X \to aX$
$A \to a$	$\cancel{Y \to bY}$	L(a) Is Empty.
$Y \to BY$	$Y \to bb$	
$Y \to BB$		
$B \to b$		

Eg*)

$S \to XS / YZ$

$X \to YX$

$Y \to YY$

$Y \to XX$

$X \to A$

$Z \to SX$

Is Empty

Eg*)

$S \to AB$	$S \to aB / bB$	$S \to abb / abbb / abbbb / bbb / bbbb / bbbbb$
$\cancel{A \to BSB}$	$\cancel{B \to aaS} / bbS / abS / baS$	
$\cancel{B \to AAS}$	$B \to bb / bbb / bbbb$	
$\cancel{A \to CC}$		L(G) Not Empty
$B \to CC$		
$\cancel{C \to SS}$		
$A \to a/b$		

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[redacted]

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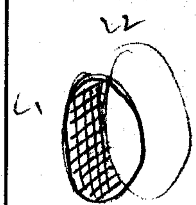
~~$CFL_1 - CFL_2$ not closed~~
~~RL - RL is closed~~

ST $L_1 - L_2 = CFL$
if $L_1 : CFL$ &
$L_2 : RL$.

$L_1 - L_2$: assume closed under difference. — ①

$L_1 - L_2 = L_1 \cap \overline{L_2}$



① ⇒ LHS is CFL
RHS is not CFL as CF languages are not closed under concatenation

∴ assumption of ① is wrong
so CFL not closed under '−'

$L_1 = CFL$
$L_2 = RL$

$L_1 - L_2 = L_1 \cap \overline{L_2}$

$\underset{CFL}{L_1} \cap \underset{RL}{\overline{L_2}}$: context free language.

⇒ If L_1:CFL, L_2:RL then closed under difference.

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ST DCFL not closed under ∪ & ∩

$L_1 \cap L_2 \Rightarrow \overline{\overline{L_1} \cup \overline{L_2}} = \overline{L}$
not DCFL

DCFL ⇒ DPDA

$L_1, L_2 \in DCFL$

$L = L_1 \cup L_2 \Rightarrow \boxed{S \to S_1 / S_2} \to$ non-deterministic
$S \to SS$
: not DCFL

(18)

S1 $L = \{ w \in \{a,b\}^* : n_a(w) = n_b(w) : w$ dont contain substring $aab \}$

$$L_2 = (a+b)^* aab (a+b)^*$$

↓ Regular language $\Rightarrow \bar{L}_2$ also RL

$$L_1 = \{ \{a,b\}^* : n_a(w) = n_b(w) \}$$

we know NOT CFL (PL fails)

$a^m b^k a^l b^j$
$(m+l = k+j)$

$L_1 \cap \bar{L}_2 = L$

($\bar{L}_2$ ↓ RL)

Case (i) L is CFL $\Rightarrow L_1$ is CFL / not true

Case (ii) L is NOT CFL $\Rightarrow L_1$ is NOT CFL / true

$\therefore L$ is not CFL